THE SOCIETY OF NAVAL ARCHITECTS AND MARINE ENGINEERS
601 Pavonia Avenue, Suite 400, Jersey City, New Jersey 07306 USA
Paper presented at the Marine Structural Inspection, Maintenance, and Monitoring Symposium
Sheraton National Hotel, Arlington, Virginia, March 18-19, 1991

Fatigue Reliability Model for Inspection, Updating and Repair of Welded Geometries

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ABSTRACT

The use of predicted fatigue crack growth behavior in the updating of the fatigue design life is investigated. From experience and experimental fatigue crack growth tests the relationship between developed crack size and remaining fatigue life can be established for groups of geometries. The probabilistic estimated fatigue design life can then be updated from inspection results, independent of type of fatigue model applied. The updating procedure is demonstrated by use of a probabilistic S-N fatigue analysis model where the effects of inspection quality and repair philosophy on the fatigue failure probability is investigated.

1. INTRODUCTION

The fatigue failure probability of a welded structure is usually estimated based on either linear elastic fracture mechanics or S-N curve fatigue life calculations. The initial estimates for the fatigue failure probability can be updated during the lifetime of the structure through inspections. With the additional information available much of the uncertainties present at design stage are removed and improved estimates of the fatigue behavior can be made.

It is demonstrated in Madsen et al[1] that the probabilistic fracture mechanics analysis is easily combined with results from inspections. If the fatigue failure probability is estimated by S-N approach the inspection results can not explicitly be applied in the reliability analysis, since the crack size is not included as a parameter in the fatigue model. The inspection results can, however, be included in the fatigue analysis if one has established a relationship between the developed crack length and the corresponding fraction of the total design life. Updated estimates of the design life can then be established from inspection results and time of inspection.

This paper shows how results from inspections can be used to update the S-N fatigue failure probability by use of full distribution reliability methods and Bayesian updating technique.

2. MINER-PALMGREN FATIGUE DAMAGE MODEL

In S-N approach, the fatigue strength is expressed in terms of a ΔS–N relation, giving the number of stress cycles N of constant stress range ΔS leading to failure:

$$N\,(\Delta S)^m = K \tag{1}$$

where K and m are empirical constants, ASCE[2]. The model is often used with a positive lower threshold on ΔS below which no damage is assumed to occur.

Usually the amplitude of the stress range is not constant over the lifetime of the structure. The calculation of the fatigue damage under varying loading can be calculated by the Miner-Palmgren model, Miner[3]. It is here assumed that the damage on the structure per load cycle is constant at a given stress level ΔS_i and that the total damage the structure is exposed to can be expressed as the accumulated damage from each load cycle at different stress range levels, independent of the sequence in which the stress cycles occur:

$$\Delta_{N_L} = \sum_{i=1}^{N_L} \frac{1}{N(\Delta S_i)} \tag{2}$$

where Δ_{N_L} is the accumulated damage over the time period with N_L load cycles. Combining these equations

$$\Delta_{N_L} = \frac{1}{K}\sum_{i=1}^{N_L} (\Delta S_i)^m \tag{3}$$

If the number of load cycles are sufficiently large the, sum can be simplified to the sum of the expected value of the load process describing the stress range:

$$\Delta_{N_L} = \frac{1}{K}\sum_{i=1}^{N_L} (\Delta S_i)^m \tag{4}$$

$$= \frac{1}{K} N_L \sum_{i=1}^{N_L} \frac{1}{N_L} (\Delta S_i)^m \approx \frac{1}{K} N_L E[(\Delta S_i)^m]$$

The failure criterion is taken as the accumulated damage exceeding the critical Miner-Palmgren damage index Δ_c, where failure is defined as crack growth through the wall thickness. Conventionally the damage index is taken as one. The design life of the structure against fatigue failure is then expressed as:

$$T_D = \frac{\Delta_c K}{\nu_0 E[(\Delta S)]} \tag{5}$$

where ν_0 is the zero crossing frequency of the loading process in unit [$year^{-1}$].

The safety margin M against fatigue failure within the lifetime T_L of the structure is described as:

$$M = T_D - T_L = \frac{\Delta_c K}{\nu_0 E[(\Delta S)^m]} - T_L \tag{6}$$

and the fatigue failure probability against through the thickness crack is:

$$P_F = P(M \leq 0) \tag{7}$$

In the modeling of the fatigue failure probability it is important that best estimate rather than conservative estimates are used for the empirical constants K and m.

3. MODEL UPDATING FROM INSPECTION

In service inspections are performed in order to assure that the existing cracks in the structure, which may be present at design stage or arise at a later stage during the service time, do not grow to critical size.

The result from an inspection is either crack detection or no detection of a crack. In the case of no crack detection in an inspection, the crack size is smaller than the smallest detectable crack size:

$$2c(N_i) \leq \lambda_d \tag{8}$$

where $2c(N_i)$ is the crack length after N_i load cycles and λ_d is the smallest detectable crack size. λ_d is dependent on inspection method and procedures applied (visual, MPI, etc.). λ_d is generally random since a crack only is, as a function of the size of the crack, detected with a certain probability. The distribution of λ_d is modeled as the probability of detection (POD) function for the inspection method applied.

In the case of crack detection the size of the detected crack λ_m is measured:

$$2c(N_j) = \lambda_m \tag{9}$$

where λ_m is the observed crack length after N_j load cycles. λ_m is also usually random since accurate determination of the detected crack length might be difficult due to possible measurement errors and errors in the interpretations of measurement signals.

To apply the inspection results in the updating of the estimated fatigue design life of the structure, it is necessary to define a model describing the relationship between the crack growth over the exposed time period and the remaining time to fatigue failure.

Tweed and Freedman[4] have established a relationship between developed crack size and remaining fatigue life for tubular joints based on experimental results. Equivalent relationships could be established for other groups of geometries. The model assumes it possible to express the design life, or the time to through the thickness crack, as a probabilistic function of the normalized developed crack length. From inspection results the relative remaining fatigue life of the component is estimated. This estimated remaining fatigue life can then be applied to update the original design life estimate. The model is extended to include the effect of crack initiation time and initial crack size.

From the experimental data, the endurance/endurance to through the thickness cracking (T/T_c) can be probabilistically described as a function of the normalized surface crack length/member thickness ($2c/t$):

$$T/T_c = h(2c/t) \tag{10}$$

where $h(\,)$ is the probabilistic function describing this relationship. By defining design life as the time to through the thickness crack the design life can be estimated from the developed crack length $2c_i$ over the time period T_i as:

$$T_D = \frac{T_i}{h(2c_i/t)} \tag{11}$$

The inspection estimated design life, T_D, is stochastic due to the probabilistic form of $h(\,)$.

The additional information available from inspections is through the definition of event margins (H) used to update the estimated design life.

$$H = T_D - \frac{T}{h(2c/t)} \tag{12}$$

For the inspection result of no crack detection the event margin is positive since the crack size is smaller than the smallest detectable crack size λ_d.

$$H = T_D - \frac{T}{h(\lambda_d/t)} \geq 0 \tag{13}$$

In the case of the detection of a crack of size λ_m the event margin is zero

$$H = T_D - \frac{T}{h(\lambda_m/t)} = 0 \tag{14}$$

The updated fatigue failure probability based on s inspections with no crack detection in the first r of these inspections and crack detection in the last $s-r$ inspections is e.g. expressed as

$$P_F = P(M \leq 0 \mid H_1 \geq 0 \cap .. \cap H_r \geq 0 \cap H_{r+1} = 0 \cap .. \cap H_s = 0) \tag{15}$$
$$= \frac{P(M \leq 0 \cap H_1 \geq 0 \cap .. \cap H_r \geq 0 \cap H_{r+1} = 0 \cap .. \cap H_s = 0)}{P(H_1 \geq 0 \cap .. \cap H_r \geq 0 \cap H_{r+1} = 0 \cap .. \cap H_s = 0)}$$

The effect of crack initiation is included in the model by the definition of an initial crack size λ_0 and a crack initiation time T_0. The event margin is then

$$H = T_D - \frac{T - T_0}{h(2c/t) - h(\lambda_0/t)} (1 - h(\lambda_0/t)) - T_0 \tag{16}$$

The uncertainties involved in the estimation of the initial crack size and the crack initiation time can be included in the probabilistic fatigue analysis by random modeling of these parameters.

In the case of repair of a detected crack λ_r at time T_{rep} the safety and event margins are modeled as

Safety Margin:

$$M = (T_{rep} + T_{Drep}) - T_L = T_{rep} + \frac{\Delta_c K_{rep}}{\nu_0 E[(\Delta S)^{m_{rep}}]} - T_L \tag{17}$$

where K_{rep} and m_{rep} are the material parameters after repair and T_{Drep} is the estimated design life after repair.

Event Margin H at time of repair:

$$H = T_D - \frac{T_{rep}}{h(\lambda_{rep}/t)} = 0 \tag{18}$$

Event Margin H for inspections at time after repair:

- No new crack detection:

$$H_{rep} = T_{Drep} - \frac{T_i - T_{rep}}{h(\lambda_{di}/t)} \geq 0 \tag{19}$$

- New crack detection:

$$H_{rep} = T_{Drep} - \frac{T_j - T_{rep}}{h(\lambda_{mj}/t)} = 0 \tag{20}$$

The dependence in the estimated design life before and after repair, T_D and T_{Drep}, is included in the analysis by defining a correlation between these time estimates directly or by introducing a correlation matrix describing the relationship among the material parameters K, K_{rep}, m, m_{rep} and between the loading processes before and after repair. The combined effect of crack initiation at design stage and also crack initiation after repair is modeled by combining the event margin defined in Eq.(16) with the event margins described above.

Event Margin H at time of repair:

$$H = T_D - \frac{T_{rep} - T_0}{h(\lambda_{rep}/t) - h(\lambda_0/t)} \left(1 - h(\lambda_{0rep}/t)\right) - T_{0rep} = 0 \tag{21}$$

Event Margin H for inspections at time after repair:

- No new crack detection:

$$H_{rep} = T_{Drep} - \frac{T_i - T_{rep} - T_{0rep}}{h(\lambda_{di}/t) - h(\lambda_{0rep}/t)} \left(1 - h(\lambda_{0rep}/t)\right) - T_{0rep} \geq 0 \tag{22}$$

- New crack detection:

$$H_{rep} = T_{Drep} - \frac{T_j - T_{rep} - T_{0rep}}{h(\lambda_{mj}/t) - h(\lambda_{0rep}/t)} \left(1 - h(\lambda_{0rep}/t)\right) - T_{0rep} = 0 \tag{23}$$

where T_{rep} is the time of repair and T_0 and T_{0rep} is the crack initiation time at design stage and after repair. λ_{rep} is the length of the repaired crack, λ_0 and λ_{0rep} are the initial crack sizes at design stage and after repair and λ_{di} and λ_{mj} are the smallest detectable crack size and the detected crack size at inspections after repair has been performed.

The effect of crack repair by grinding compared to welding can be modeled by assuming a longer crack initiation period after repair T_{0rep} using the grinding method and by applying equivalent material parameters before and after repair. A more general situation including inspections of several locations with potential crack growth can be considered applying the same updating procedure. Dependence among basic variables referring to different locations, as loading process and material parameters must then, however, be included in the model.

4. RELIABILITY METHOD

The reliability method applied for evaluating the failure probability is the first order reliability method (FORM). This method is reviewed thoroughly in Madsen et al[5] and only a short summary is given here.

In full distribution reliability methods the basic stochastic variables $\bar{Z}$ definding the safety margins are transformed into a set of independent and standardized normal variables $\bar{U} = T(\bar{Z})$, where $T(\bar{Z})$ is this transformation. The limit state surface in U-space divides this space into a safe set and a failure set, and the failure probability is the probability contence of the failure set $M(\bar{Z}) \leq 0$. In the first order reliability method the limit state surface is approximated by a tangent hyperplane through the point on the limit state surface closest to the origin, defined as design point.

The evaluation of the parallel-system defined in Eq.15 is approximated by a linearization through the joint design point for the safety and event margins and by use of the multinormal distribution. The failure probability of the parallel-system can then be approximated as,

$$P_F = \bar{\Phi}(-\bar{\beta}; \bar{\rho}) \tag{24}$$

where $\bar{\beta}$ is the vector of the first order reliability indexes for the safety and event margins of the parallel-system, $\bar{\rho}$ is the correlation matrix for these margins and $\bar{\Phi}$ is the standardized multinormal distribution. A more detailed description of this approximation and the the modeling of the event margins with equality constraints is given in Madsen[6].

The evaluation of the fatigue failure probabilities in the numerical examples were carried out by applying the computer program PROBAN, Olesen and Skjong[7].

5. NUMERICAL EXAMPLE

A probabilistic fatigue analysis of a tubular joint is performed. The distribution of the parameters involved in the analysis are choosen to exemplify the method and do not necessarily represent a real life situation.

The surface crack development data presented in Tweed and Freedman[4] are applied in the probabilistic analysis. These data describe the probabilistic endurance/endurance to through the thickness cracking as a function of the normalized surface crack length/member thickness, see Figure 1. From a regression analysis of these data, Hanna and Karsan[8] estimated the mean and standard deviation of the relative remaining joint fatigue life to be:

$$E[h(2c/t)] = 0.383\,(2c/t)^{(1\;3\;0)} \tag{25}$$

$$SD[h(2c/t)] = 0.143\,(2c/t)^{(1\;10.6)}$$

The probabilistic distribution describing the relative remaining joint life will necessarily be bounded by 0 and 1, and a Beta distribution with these bounds and the expressions for the mean and standard deviation given above is applied to describe the distribution of the relative remaining joint fatigue life as a function of the crack length,

$$f_{2c/t}(x) = \frac{1}{(b-a)^{(s-1)} \int_0^1 u^{r-1}(1-u)^{s-r-1}\,du} (x-a)^{r-1}\,(b-x)^{s-r-1} \tag{26}$$

where a and b are the lower and upper bounds and the parameters r and s are estimated from the expressions of the mean and standard deviation

$$E[x] = a+(b-a)\,\frac{r}{s} \tag{27}$$

$$SD[x] = (b-a)\,\frac{r}{s}\,\frac{\sqrt{(s-r)}}{\sqrt{r(s+1)}}$$

The updating of the fatigue analysis based on inspection results can be performed with the stress range distribution resulting from a detailed uncertainty modeling of the environmental conditions, load model, global response and stress calcuation. It is, however, extremely time saving to calibrate a stress range distribution with a smaller number of random variables. A Weibull distribution is selected

$$F_{\Delta S}(s) = 1 - \exp[-(\frac{s}{A})^B] \tag{28}$$

where the Weibull parameters A and B are defined as random distribution parameters in the reliability analysis to include the uncertainties involved in estimation of the longterm stress distribution. A joint normal distribution for $(lnA, 1/B)$ is selected, typically representing the loading condition for a North Sea jacket structure, Madsen and Sorensen[9].

$$E[lnA] = -1.60,\ \sigma[lnA] = 0.22,\ \rho[lnA, 1/B] = -0.79 \tag{29}$$

$$E[1/B] = 1.31,\ \sigma[1/B] = 0.14$$

The m'th moment of the stress range is also random due to the random distribution parameters.

$$E[(\Delta S)^m] = A^m\,\Gamma(1 + \frac{m}{B}) \tag{30}$$

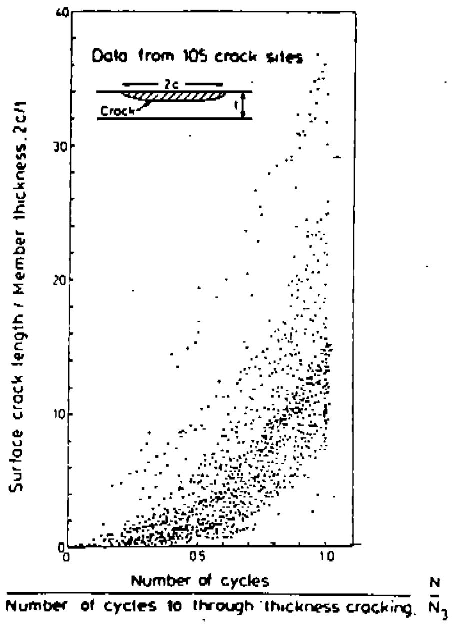

Fig.1: Database for surface crack development in tubular joint fatigue tests, Tweed and Freeman[4].

The inspection quality is modeled by treating the detectable crack length λ_d as a stochastic variable. The probability of detection curve POD is assumed to be of exponential form

$$P_{detect} = F_{\lambda d}(\lambda) = 1 - exp(-\frac{\lambda}{\alpha}) \tag{31}$$

The parameter α adjusts the crack detectability of the inspection. The numerical example is based on an α=6.21, α=18.63 and α=55.89 modeling an MPI inspection with 80% probability of detecting a crack of length 10 mm, 30 mm and 90 mm. To include confidence bounds on the POD curve the parameter α can be modeled as a random variable.

The S-N curves are founded on statistical analysis of appropriate experimental data. They consist of linear relationships between $\log_{10}\Delta S$ and $\log_{10}N$. The design curve is defined as the mean minus two standard deviations of $\log_{10}N$. Best estimate values rather than conservative values should be chosen on the material parameters in a probabilistic analysis and randomized mean values are here applied. Department of Energy suggest the following mathematical form of the design S-N curve for tubular joints in seawater with cathodic protection

$$\log_{10}(N) = \log_{10}(K) - m\log_{10}(\Delta S) \tag{32}$$

$$= 12.16 - 3.0\log_{10}(\Delta S) - \frac{m}{4}\log_{10}(t/32)$$

where the last term is the thickness correction factor and t is the thickness in mm through which the potential crack will grow. The $\log_{10}(K)$ was here modeled as $N(12.66, 0.24)$. The damage measure, Δ_c, is modeled with a coefficient of variation of 0.20 to include the uncertainties involved in determing the Miner sum at through the thickness crack.

The probability of fatigue failure as a function of years in service based on a S-N fatigue analysis is shown in Figure 2. No initial crack defect or crack initiation period were assumed. The results are expressed in terms of a reliability index β_R which is uniquely related to the failure probability as

$$\beta_R = -\Phi^{-1}(P_F) \tag{33}$$

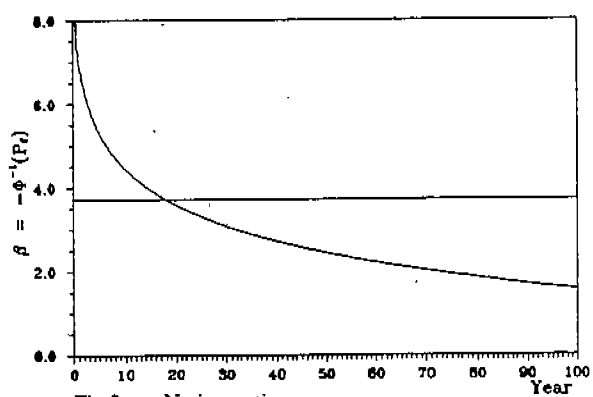

Fig.2: No inspections

From inspections more information about the fatigue behavior will be gained and some of the uncertainties in the modeling will be reduced. Figure 3, 4 and 5 show how the reliability index changes based on MPI inspections with α=6.21, α=18.63 and α=55.89 and no crack detections. The time of inspections are based on a maximum permitable failure probability of 10^{-4} over the lifetime of the structure. The figures show that a higher inspection quality gives more confidence in the inspection results, higher estimated reliability of the structure and then longer inspection intervals.

Figure 6 shows the change in the reliability index based on inspection with detection of a crack of size 16 mm and 50 mm after 18 years of service. For both observations, we are seeing a drastic reduction of the estimated reliability index, indicating a high probability for a through the thickness crack within the lifetime of the structure, unless a repair is performed.

Figure 7 and 8 indicate the effect of weld repair and grind repair of a detected 50 mm long crack, with no crack detection at the first inspection after repair. Weld repair is modeled by assuming independent, identically distributed material parameters before and after repair, with no crack initiation period. Grind repair is modeled by assuming identical material parame-

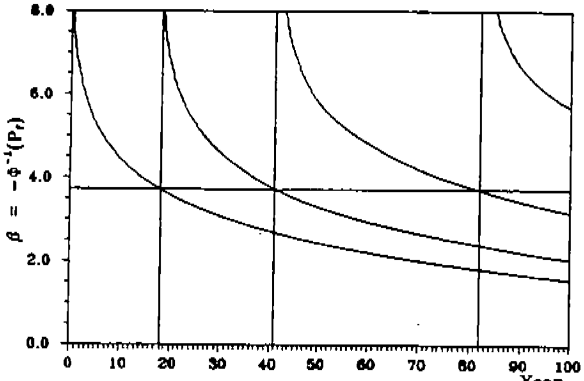

Fig.3: No crack detection for inspection with 80% probability of detecting a crack of length 10 mm.

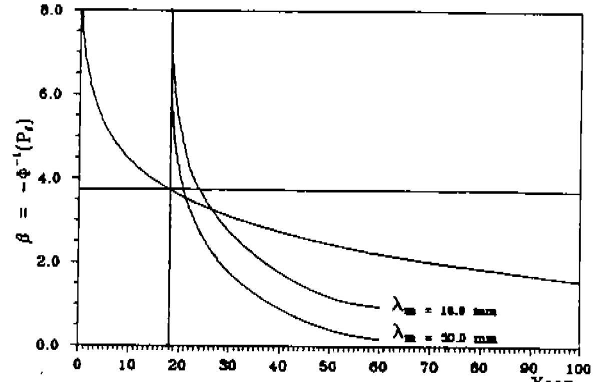

Fig.6: Inspection with detection of crack of length 16 mm and 50 mm after 18 years of service.

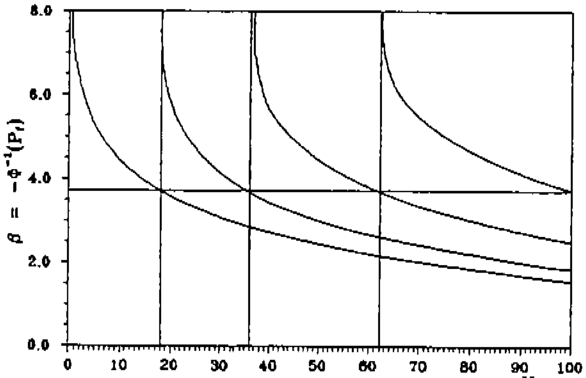

Fig.4: No crack detection for inspection with 80% probability of detecting a crack of length 30 mm.

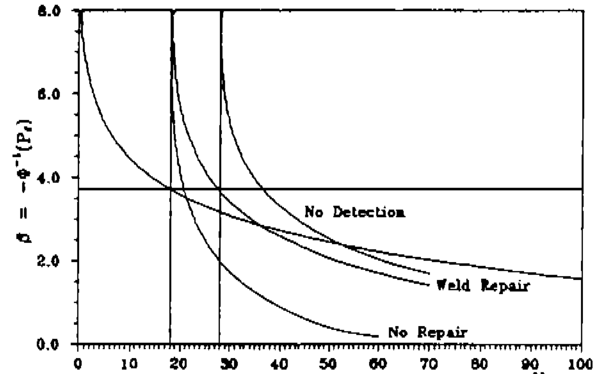

Fig.7: Inspection of weld repaired detected crack of length 50 mm after 18 years of service, with no new crack detection for inspection with 80% probability of detecting a crack of length 10 mm.

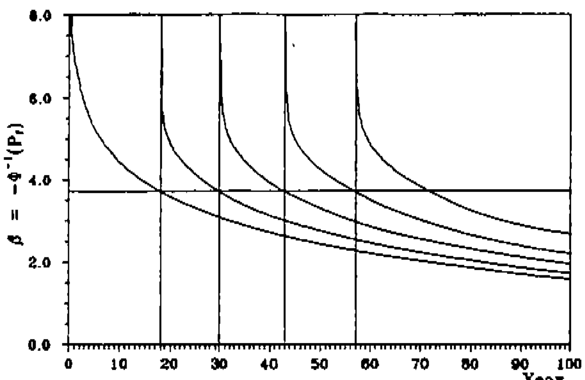

Fig.5: No crack detection for inspection with 80% probability of detecting a crack of length 90 mm.

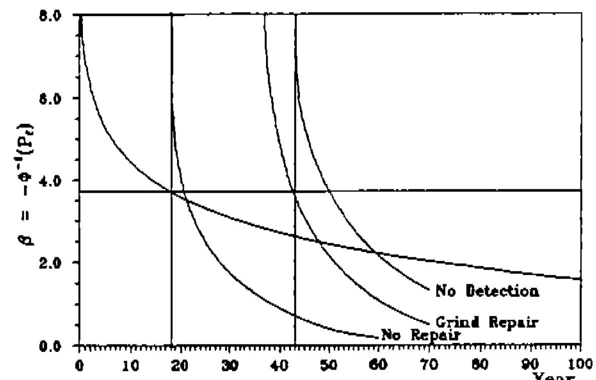

Fig.8: Inspection of grind repaired detected crack of length 50 mm after 18 years of service, with no new crack detection for inspection with 80% probability of detecting a crack of length 10 mm.

ters before and after repair and a Lognormal distributed crack initiation period with mean value 10 years and a coefficient of variation equal to 0.5. The crack initiation period is in addition modeled as a function of the stress range by applying a negative correlation between stress range process and the crack initiation time.

The results for grind repair are here highly dependent on the choice of crack initiation period. The reliability level of grind repair will after some time fall below the level of weld repair due to the assumption of identical material parameters before and after repair in the case of grind repair.

6. CONCLUSION

An analytical procedure has been developed to incorporate results from inspections and repair operations into S-N cuve based evaluations of fatigue reliability. The procedure is founded on an experimentally based relationship between surface crack length and the cyclic strains required to cause complete separation of the weld.

Numerical analyses of an example tubular joint in a North Sea platform indicate the critical importance of the inspection method and procedure in providing a basis for determining inspection intervals. Inspection intervals are reduced by a factor of two when the 80 percent POD a crack of length 10 mm is changed to 90 mm. There is little definitive information available to define reasonable POD curves for in-service structures using various practical inspection methods and procedures. This is an important area for additional research.

Similarly, the numerical results indicate the importance of assumptions regarding the effectiveness of repairs on inspection intervals and fatigue reliability. Again, definitive information for characterizing the effectiveness of various types of repairs (particularity those made underwater) does not exist. This is also an important area for additional research.

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