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Application of Reliability Assessment Methods to Marine Frame Structures Based on Ultimate Strength Analysis

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ABSTRACT

This paper presents recent development in the reliability assessment of marine frame structures which are modelled as relatively stiff frames and subjected to quasi-static extreme loads, based on ultimate collapse analysis. At first, a linearized failure condition of the section is introduced which takes into account combined load effects of bending moment, axial force and shearing force on the various failure modes. The failure criterion greatly facilitates generation of the safety margins and calculation of the failure probabilities. Structural failure is defined as production of large deflection due to collapse. Second, the so-called branch-and-bound method combined with the heuristic operations is applied to select the probabilistically dominant failure modes, which save the computation efforts to perform the reliability analysis of large-scale structures. Finally, the proposed methods are applied to an offshore jacket platform with brittle elements and a transverse structure of ships under some notional load conditions. Through the numerical examples, the effects of brittle elements, combined loads and loading conditions on probabilistic properties of ultimate collapse of marine frame structures are investigated.

INTRODUCTION

Various types of marine structures including drilling rigs, platforms, etc. have been constructed. They are required to have better operating performance in the severer state of sea and weather, and as a recent trend, they are becoming larger in size and more complex. For those marine structures which have little experience of service, a relative measure of their safety for comparison with the notional safety of existing structures can only be found by using reliability analysis methods. Many studies have been made of reliability analysis of marine structures, as reviewed from the view point of design philosophy(1). However, there remain many works to be done for large structures which have too many failure modes to identify all of them for estimating system reliability based on ultimate collapse analysis(2-11).

This paper presents recent development in the reliability assessment of marine structures which are modelled as relatively stiff frames and subjected to quasi-static extreme loads, based on ultimate collapse analysis. Ultimate collapse is evaluated by using a linearized failure condition of the section under the combined effect of bending moment, shearing force and axial force to generate the safety margins, using a matrix method. Probabilistically dominant collapse modes are selected by applying the so-called branch-and-bound method combined with the heuristic operations. These methods are applied to the following marine structures:

(1) an offshore jacket platform with brittle elements in which the bending moment and axial force dominate the failure criterion.

(2) a transverse structure of ships in which the combined effect of the bending moment, shearing force and axial force determines the plasticity condition and to which some notional load conditions are applied.

Through the numerical examples, the effects of brittle members, combined loads and loading conditions on the probabilistic properties of ultimate collapse of marine frame structures are investigated.

GENERATION OF STRUCTURAL FAILURE MODES FOR PLANE FRAME STRUCTURE UNDER COMBINED EFFECT OF BENDING MOMENT, SHEARING FORCE AND AXIAL FORCE

Consider a frame structure whose elements are uniform and homogeneous and to which only concentrated loads and moments are applied. In such a frame structure, critical sections where plastic nodes may form are the joints of the elements and the places at which the concentrated loads are applied. The following description is concerned with the case when various failures occur under combined load effects of bending moment, shearing force and axial force. In the case of plastic collapse, behaviour of members is approximated and structural analysis is performed by combining a plastic node method and a matrix method based on the displacement method(11-19).

Derivation of Reduced Stiffness Matrixes and Equivalent Nodal Forces

Let $x_t = (F_{xi}, F_{yi}, M_{zi}, F_{xj}, F_{yj}, M_{zj})^T$ and $\delta_t = (v_{xi}, v_{yi}, \theta_{zi}, v_{xj}, v_{yj}, \theta_{zj})^T$ denote the nodal force and displacement vectors of the unit element i, j, e.g., the element number t in the local coordinate system shown in Fig. 1(a).

When the interaction of bending moment, shearing force and axial force is considered, the yielding condition of the deep girder consisting of the transverse ring of a tanker is usually given by a nonlinear and asymmetric surface with regard to internal forces, as shown by thin lines in Fig. 2. However, in order to facilitate the treatment of yield condition, the yield surface is approximated by a linearized function resulting in underestimation of the strength of the member, as shown by thick lines in Fig. 2. Then, plasticity condition of a cross section is given in the following form:

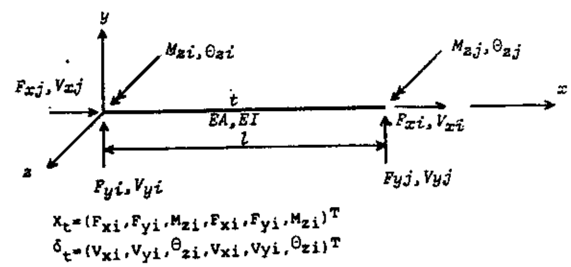


Fig. 1(a) Nodal forces and displacements of the elasto-plastic element

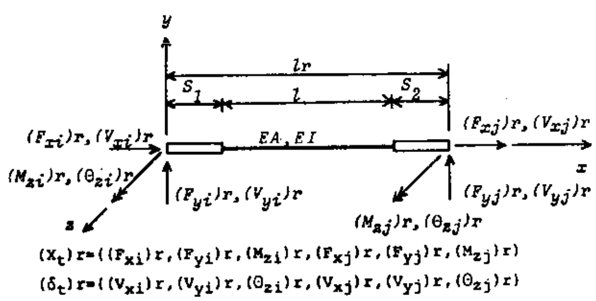


Fig. 1(b) Nodal forces and displacement of the elasto-plastic element with rigid bodies at both ends

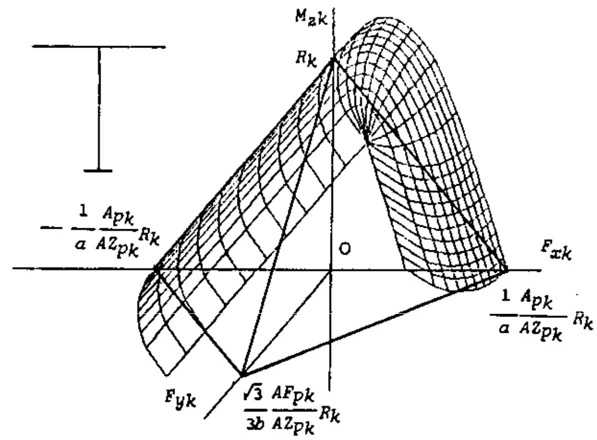


Fig. 2 Linearized plasticity condition considering the interaction of bending moment, shearing force and axial force

$$F_k = R_k - C_k^T X_t = 0 \qquad (k=i, j) \tag{1}$$

In Eq. (1), R_k is the reference strength of the element end k, which is taken to be a fully plastic moment, i.e., $R_k = \sigma_{yk} AZ_{pk}$ (AZ_{pk} : plastic section modulus of element end k, σ_{yk} : yield stress).

C_k^T is a factor determined by the dimension of element k. Particularly, the expression for the effect of bending moment, shearing and axial force upon the plasticity condition is given as follows :

$$C_i^T = \left(a\cdot\frac{AZ_{pi}}{A_{pi}}\,\mathrm{sign}(F_{xi}),\ b\cdot\frac{\sqrt{3}AZ_{pi}}{AF_{pi}}\,\mathrm{sign}(F_{yi}),\ c\cdot\mathrm{sign}(M_{zi}),\ 0,\ 0,\ 0\right) \tag{1a}$$

$$C_j^T = \left(0,\ 0,\ 0,\ a\cdot\frac{AZ_{pj}}{A_{pj}}\,\mathrm{sign}(F_{xj}),\ b\cdot\frac{\sqrt{3}AZ_{pj}}{AF_{pj}}\,\mathrm{sign}(F_{yj}),\ c\cdot\mathrm{sign}(M_{zj})\right) \tag{1b}$$

where

A_{pk} : cross-sectional area of the element end

AF_{pk} : effective sectional area of the element end for shearing force

sign(.) : sign of (.)

a, b : coefficient of axial force and shearing force effect, respectively

c : coefficient of bending moment effect

The plasticity condition (1) reduces to (i) in case of a=b=0 and c=1 : the well-known plasticity condition subjected solely to bending moment, (ii) in case of a≠0, b=0 and c=1 : the plasticity condition considering the interaction of bending moment and axial force, and (iii) in case of a≠0, b≠0 and c=1 : the condition considering the interaction of bending moment, shearing force and axial force.

For the case of an offshore jacket platform in which the bending moment and axial force dominate the failure criterion, the following values are adopted(11):

$$a = 1,\ b = 0,\ c = 1 \tag{2}$$

On the other hand, for the reliability assessment of a transverse ring of a ship in which the combined effect of bending moment, shearing force and axial force determines the plasticity condition, the following values are used:

$$a = 1,\ b = 0.5,\ c = 1 \tag{3}$$

The failure condition of the element which behaves as a brittle material, such as buckling collapse of beam-columns with initial imperfection, punching shear failure of tubular joints and brittle fracture of welding joints with fatigue cracks, is also represented by Eq.(1), where the coefficients a, b and c are given in the following:

In case of the buckling collapse:

$$a = \sigma_{yk}/\sigma_{ck}$$

$$\sigma_{ck} = \frac{1}{2}\left(\sigma_{yk}+\sigma_E+\frac{w_0}{S}\sigma_E\right)\cdot\left(1-\sqrt{1-(4\sigma_E\sigma_y)/\left(\sigma_y+\sigma_E+\frac{w_0}{S}\sigma_E\right)^2}\right)$$

w_0 : initial imperfection

σ_E : Euler's buckling stress

S : core radius

$$b = 0$$

$$c = \frac{AZ_{pk}}{AZ_{ek}} = f_k$$

f_k : shape factor

(4)

In case of the punching shear failure:

$$a = \sqrt{3}(t_k/T_k)\sin\theta_k$$

t_k, T_k : brace and chord thickness

θ_k : brace angle(measured from chord)

$$b = 0$$

$$c = \sqrt{3}(t_k/T_k)\sin\theta_k$$

(5)

In case of the brittle fracture:

$$a = K_t$$

K_t : stress concentration factor

$$b = 0$$

$$c = K_t f_k$$

(6)

Next, the behaviour of yielded section follows the plasticity theory because the perfectly elasto-plastic (or elasto-brittle) relationship has been employed into the plasticity condition. The relation between the nodal force vector $\mathbf{X}_t$ and the displacement vector $\boldsymbol{\delta}_t$ of an element including plastic nodes is derived by using plasticity theory as follows (11-19):

$$X_i = k_i^{(p)}\delta_i + \bar{X}_i^{(p)} \tag{7}$$

where

$k_i^{(p)}$: reduced element stiffness matrix

$\bar{X}_i^{(p)}$: equivalent nodal force vector

The explicit forms of $k_i^{(p)}$ and $\bar{X}_i^{(p)}$ are given as follows:

(a) In case of an elastic element:

$$k_i^{(p)} = k_i \quad (\ k_i : \text{elastic element stiffness matrix})$$
$$\bar{X}_i^{(p)} = 0 \tag{8a}$$

(b) In case of failure at left-hand end: (for ductile element)

$$k_i^{(p)}(=k_i^L) = k_i - k_iC_iC_i^Tk_i/(C_i^Tk_iC_i)$$
$$\bar{X}_i^{(p)}(=\bar{X}_i^L) = R_ik_iC_i/(C_i^Tk_iC_i) \tag{8b}$$

(for brittle element)

$$k_i^{(p)} = 0, \qquad \bar{X}_i^{(p)} = 0 \tag{8b$'$}$$

(c) In case of failure at right-hand end: (for ductile element)

$$k_i^{(p)}(=k_i^R) = k_i - k_iC_jC_j^Tk_i/(C_j^Tk_iC_j)$$
$$\bar{X}_i^{(p)}(=\bar{X}_i^R) = R_jk_iC_j/(C_j^Tk_iC_j) \tag{8c}$$

(for brittle element)

$$k_i^{(p)} = 0, \qquad \bar{X}_i^{(p)} = 0 \tag{8c$'$}$$

(d) In case of failure at both ends: (for ductile element)

$$k_i^{(p)}(=k_i^{LR})=k_i-[H]^T[G^{-1}][H]$$

$$\bar{X}_i^{(p)}(=\bar{X}_i^{LR})=[H]^T[G^{-1}]\begin{Bmatrix}R_i\\R_j\end{Bmatrix}$$

$$[G^{-1}]=\begin{bmatrix}C_i^Tk_iC_i & C_i^Tk_iC_j\\C_j^Tk_iC_i & C_j^Tk_iC_j\end{bmatrix}^{-1},\quad [H]=\begin{bmatrix}C_i^Tk_i\\C_j^Tk_i\end{bmatrix} \tag{8d}$$

(for brittle element)

$$k_i^{(p)}=0,\qquad \bar{X}_i^{(p)}=0 \tag{8d)'$$

Consider an element with rigid bodies at both ends which is idealized for a transverse ring of a ship . Let $(X_t)_r$ and $(\delta_t)_r$ respectively denote the nodal force and displacement vectors of the outside of unit element i, j with rigid bodies whose lengths are s_1 and s_2 , as shown in Fig. 1(b). By using transformation matrix τ_t : $\delta_t = \tau_t (\delta_t)_r$,

$$\tau_t=\begin{bmatrix}1&0&0&0&0&0\\0&1&s_1&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&-s_2\\0&0&0&0&0&1\end{bmatrix} \tag{9}$$

and the relation between X_t and δ_t for the elasto-plastic element, the following relation is obtained:

$$(X_t)_r=(k_t^{(p)})_r(\delta_t)_r+(\bar{X}_t^{(p)})_r \tag{10}$$

where

$$(k_t^{(p)})_r=\tau_t^T k_t^{(p)}\tau_t$$
$$(\bar{X}_t^{(p)})_r=\tau_t^T\bar{X}_t^{(p)}$$

Generation of Safety Margins and Structural Failure Criterion

Consider a plane frame structure with n elements and at most $3l$ loads applied to its l nodes. The failure criterion of the i-th elasto-plastic element end is given by

$$Z_i=R_i-C_i^T X_t\leq 0 \tag{11}$$

Structural failure of a frame structure is defined as occurrence of large nodal displacement due to plastic collapse. A criterion for structural failure is given as in the following manner. When any one element end yields, the internal forces are redistributed to the element ends. Similarly when some element ens r_1, r_2, ..., r_{p-1} have failed, stress analysis is performed once again and the stiffness equation of the element is replaced by the corresponding reduced one, e.g., Eq. (7) or Eq.(10). The reduced element stiffness matrixes are evaluated for all the failed elements, and they are assembled to have the total structure stiffness matrix:

$$(K^{(p)})\ (d)=(L)+(R^{(p)}) \tag{12}$$

where

(d) : total nodal displacement vector referred to the global coordinate system

$(K^{(p)})=\sum_{k=1}^{n}T_k^T\tau_k^T K_k^{(p)}\tau_k T_k$

: reduced total structure stiffness matrix

T_k : transformation matrix

(L) : vector of external loads

$(R^{(p)})=-\sum_{k=1}^{n}T_k^T\tau_k^T\bar{X}_k^{(p)}$

: equivalent nodal force vector referred to the global coordinate system

Finally, the nodal force vector X_t of the t-th element is given by

$$X_t=b_t^{(p)}[(L)+(R^{(p)})]+\bar{X}_t^{(p)} \tag{13}$$

where $b_t^{(p)}=\tau_t T_t[(K_t^{(p)})]^{-1}$

$[(K_t^{(p)})]^{-1}$: the matrix formed by extracting the rows corresponding to the t-th element from the matrix $[(K^{(p)})]^{-1}$

Now that the element ends r_1, r_2, ... , and r_{p-1} have failed, the safety margin of the surviving element end i (element number t) is obtained by substituting Eq. (13) into Eq. (11):

$$Z_i^{(p)}=R_i+C_i^T(b_t^{(p)}\sum_{k=1}^{n}T_k^T\tau_k^T\bar{X}_k^{(p)}-\bar{X}_t^{(p)})-C_i^T b_t^{(p)}(L) \tag{14}$$

$$=R_i+\sum_{k=1}^{p-1}a_{ir_k}^{(p)}R_{r_k}-\sum_{j=1}^{3l}b_{ij}^{(p)}L_j \tag{15}$$

where $a_{ir_k}^{(p)}$ and $b_{ij}^{(p)}$ are the coefficients resulted from resolution of the vectors into their components.

Occurrence of large nodal displacements due to the plastic collapse is determined by investigating the property of the total structure stiffness matrix $[K^{(p)}]$. For example, when the element ends up to some specified number , e.g., element ends r_1, r_2, ... , r_{pq} , have failed and the reduced total structure stiffness matrix $[K^{(pq)}]$ satisfies the following condition, structural failure results:

$$| \, [(K^{(p_q)})_r] \, | \, / \, | \, [(K^{(0)})_r] \, | \leq \varepsilon \tag{16}$$

where superscripts (p_q) and (0) are used to denote the p_q-th failure stage and the elastic condition, respectively. ε is the specified constant for determining the plastic collapse. The sequence of the failed element ends to produce structural failure, e.g., r_1, r_2, ..., r_{p_q} is called a complete failure path.

By using the above equation, a criterion of structural failure is given by

$$Z_{r_p}^{(p)} \leq 0 \qquad (p = 1,2,\ldots,p_q) \tag{17}$$

If there are any failed element ends r_p, which have their coefficients $a_{r_{p_q} r_p}^{(p_q)}$ equal to zero in the safety margin $Z_{r_{p_q}}^{(p_q)}$ of the last yielded element end e.g.,

$$a_{r_{p_q} r_p}^{(p_q)} = 0 \tag{18}$$

they are the redundant element ends which do not directly contribute to occurrence of the plastic collapse. Alternatively, those element ends are called essential without which no plastic collapses are formed. A minimum set of plastic nodes is a failure path including no redundant plastic nodes. A failure mode is a set of plastic nodes comprising the minimum set.

In summary, the the plasticity condition of the element end under the combined loads has been approximated by a linear surface given by Eq. (1), and the safety margin of the element end has also been expressed as a linear combination of the strengths of the element ends and the applied loads. Consequently, reliability analysis is greatly facilitated when the strengths and the loads are normal random variables.

AUTOMATIC SELECTION OF PROBABILISTICALLY DOMINANT FAILURE PATHS

There are too many failure paths in a highly redundant structure (14,17,19) to generate all of them, which necessitates a proceduree for selecting only the probabilistically significant failure paths. Efficient methods by using a branch-and-bound technique have been proposed (14,16,17,19) and this paper adopts the procedure given in the following.

Branching Operations

These operations are to select the plastic nodes such that stochastically dominant failure paths may be obtained. An element end (called here node for simplicity) is selected as a plastic node at the p-th failure stage based on two-dimensional joint probability (so-called two dimensional branchings). The node to be selected at the p-th failure stage is given by

$$P[Z_{r_1}^{(1)} \leq 0] = \max_{i_1 \in I_1} P[Z_{i_1}^{(1)} \leq 0] \quad \text{for } p=1 \tag{19}$$

$$P[(Z_{r_1}^{(1)} \leq 0) \cap (Z_{r_p}^{(p)} \leq 0)] = \max_{i_p \in I_p} P[(Z_{r_1}^{(1)} \leq 0) \cap (Z_{i_p}^{(p)} \leq 0)] \quad \text{for } p \geq 2 \tag{20}$$

where I_p : the set of nodes i_p to be selected at the p-th failure stage

$Z_{i_1}^{(1)}$: safety margin of node i_1 at the first failure stage, i.e., when no plastic nodes exist in the structure

$Z_{i_p}^{(p)}$: safety margin of node i_p at the p-th failure stage, i.e., after formation of plastic nodes at the sections r_1, r_2, ..., $r_{p-1} (p \geq 2)$

The joint probability is calculated with Hermite polynominal expansion method(20). By repeating the selecting process, a sequence of plastic nodes to form a plastic collapse, e.g., a complete failure path (r_1, r_2, ..., and r_{p_q}) is found.

The lower and upper bounds, $P_{fp(q)(L)}^{(p)}$ and $P_{fp(q)(U)}^{(p)}$, of the probability $P_{fp(q)}^{(p)}$ of a particular partial failure path q up to the p-th $(p \geq 2)$ failure stage is evaluated by the following formulas :

$$P_{fp(q)(L)}^{(p)} \leq P_{fp(q)}^{(p)} = P[\bigcap_{i=1}^{p} (Z_{r_i(q)}^{(i)} \leq 0)] \leq P_{fp(q)(U)}^{(p)} \tag{21}$$

$$P_{fp(q)(U)}^{(p)} = \min_{j \in \{2,\ldots,p\}} P[(Z_{r_1(q)}^{(1)} \leq 0) \cap (Z_{r_j(q)}^{(j)} \leq 0)] \tag{22}$$

$$P_{fp(q)(L)}^{(p)} = \max \{0, 1 - P[S_1] - \sum_{i=2}^{p} \min_{j \in \{1,2,\ldots,i-1\}} P[\bar{S}_j \cap S_i]\} \tag{23}$$

In equation (23), S_i's designate the non-failure events $Z_{r_i(q)}^{(i)} \geq 0$ ($i=1,2,\ldots,p$) rearranged in the decreasing order of probabilities(20):

$$P[S_1] \geq P[S_2] \geq \cdots \geq P[S_p] \tag{24}$$

Further, the following bound(21) is also applicable when all the correlation coefficients are non-negative, i.e.,

$$P_{fp(q)(L)}^{(p)} = \int_{-\infty}^{\infty} \phi(t) \cdot \prod_{j=1}^{p} \Phi((-\beta_j - t\lambda_j)/(1-\lambda_j^2)^{1/2})\, dt$$

$$\lambda_j = \{\min_{i \neq j} (\rho_{ij})\}^{1/2}, \quad (\rho_{ij}) \geq 0 \qquad (25)$$

β_j is the reliability index at the j-th failure stage, ϕ and Φ are standard normal probability density function and standard normal probability distribution function, respectively.

Eqs. (23) and (25) need the safety margins at all the failure stages. It should be noted here that the lower bound is calculated only when a complete failure path is found.

The maximum P_{fpM} of the lower bounds of the selected complete failure path probability is calculated:

$$P_{fpM} = \max_q P_{fp(q)(L)}^{(p_q)} \qquad (26)$$

P_{fpM} is updated when a new complete failure path is found and its failure probability is larger than the previous P_{fpM}. The branching operations are terminated when no nodes are left for selection.

Bounding Operations

These operations are to select the nodes to be discarded. The nodes deleted at the p-th failure stage are those :

$$P[Z_{i_1}^{(1)} \leq 0]/P_{fpM} < 10^{-\gamma_1} \quad \text{for } p=1 \qquad (27)$$

$$P[P_{fp(q)(U)}^{(p)}]/P_{fpM} < 10^{-\gamma_2} \quad \text{for } p \geq 2 \qquad (28)$$

where γ_1 and γ_2 are the specified constants.

From this, it is concluded that neglected failure paths are those which have the failure probabilities smaller than $10^{-\gamma_i} P_{fpM}$ $(i=1,2)$.

The probability of occurrence P_{fq} for the failure mode, i.e., the set of plastic nodes to produce structural failure, corresponding to the selected failure path is estimated by

$$P_{f_q} = P[Z_{r_{p_q}}^{(p_q)}{}_{(q)} \leq 0] \qquad (29)$$

for the structure consisting of ductile members, or

$$P_{f_q} = \min_{p \in \{1,2,\ldots,p_q\}} P[Z_{r_n(q)}^{(p)} \leq 0] \qquad (29)'$$

for the structure with brittle members.

Heuristic Operations

The number of branchings become enormous for a large scale structure with high degree of redundancy, even though the branch-and-bound method is applied. To reduce the computational effort, the following heuristic operations are applied. First, the reliability assessment is performed of some structural divisions which are presumed to be critical. The lower bounds of the resulting complete failure path probabilities are used as the reference value P_{fpM} for bounding operations. Second, the set of nodes for branching is restricted to the nodes which satisfy the monotony conditions of the failure probabilities. That is

$$I_{p_1} = \{i_p \mid P[Z_{i_p}^{(p)} \leq 0] \leq a_1 P[Z_{r_{p-1}}^{(p-1)} \leq 0]\} \quad \text{for } p \geq 2 \qquad (30)$$

Third, the contribution of the first plastic node is taken into account:

$$I_{p_2} = \{i_p \mid a_{i_p r_1} \geq a_2\} \qquad (31)$$

Fourth, the number of branchings from one failure stage is restricted to a specified number a_3.

APPLICATION TO MARINE FRAME STRUCTURES

The above method is applied to a jacket-type offshore platform with brittle members and a transverse structure of three types of ships. The former is given mainly to show the property on behaviour of the structure with brittle members and the latter is chosen to study the combined load effect and loading condition on the probabilistic collapse analysis. All the random variables are assumed to be normally distributed.

Jacket Structure

A jacket-type offshore platform shown in Fig. 3 is considered. The dimensions and strengths of members are shown in Tabe I and it is assumed that the strengths of the nodes in the same elements are completely dependent normal random variables. The notional mean values of the extreme wave loads are given in Fig. 3 and their coefficients of variation are 0.30. The brace elements are assumed to behave like brittle or ductile truss elements. The plasticity condition takes account of the combined load effect of bending moment and axial force (a=1,b=0 , and c=1). The results are listed in Tables II and III

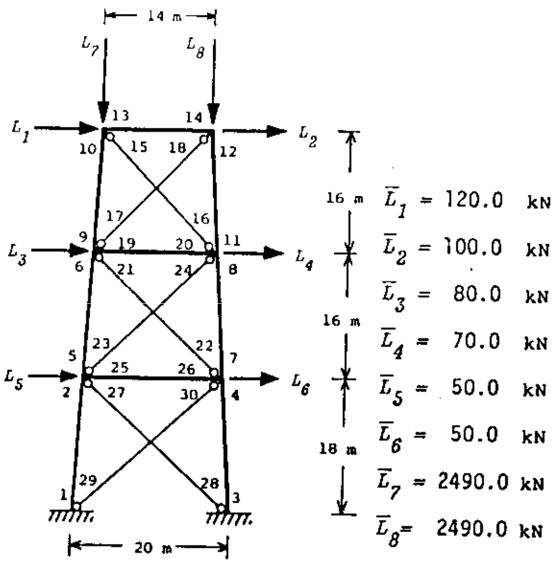


Fig. 3 Jacket-type structure

Table I Numerical data of the jacket-type structure

Element end number	*Cross sectional area* $A_{pi} = A_i$ m^2	*Moment of inertia* I_i m^4	*Mean value of reference strength* $\bar{R}_i$ kNm
1, 2 3, 4	5.37×10^{-2}	2.188×10^{-3}	2536.6
5, 6 7, 8 9, 10 11, 12	3.73×10^{-2}	1.055×10^{-3}	1467.9
13, 14 19, 20	4.68×10^{-2}	1.660×10^{-3}	2062.3
15, 16 17, 18	2.10×10^{-3}	-	20.29
21, 22 23, 24	3.40×10^{-3}	-	39.63
25, 26	4.85×10^{-2}	1.782×10^{-3}	2174.8
27, 28 29, 30	9.30×10^{-3}	-	183.5

Young's modulus E = 210 GPa

Mean value of yield stress $\bar{\sigma}_{yi}$ = 276 MPa

Correlation coeff. $\rho_{L_iL_j} = 0.0$

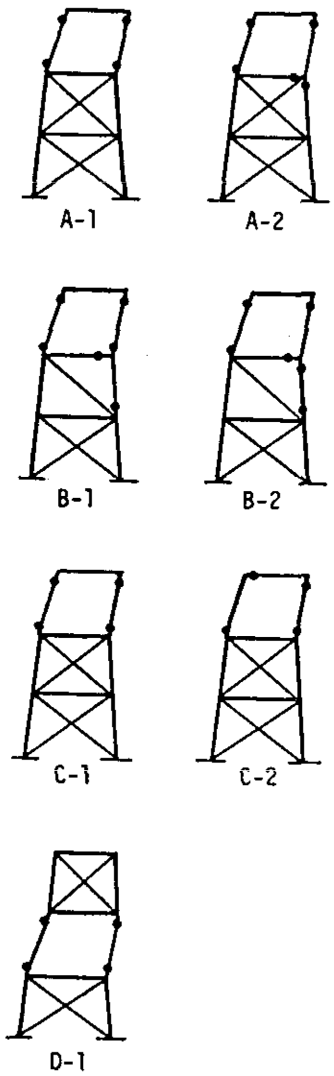
The strengths of the element ends in the same elements are completely dependent normal random variables.

Table II Calculated results of the jacket-type structure with brittle braces

$\gamma_1 = \gamma_2 = \gamma = 3.0$, $\varepsilon^{\#} = 0.001$, $CV_{R_i}/CV_{L_j} = 0.15/0.30$

Type	Failure path (Brittle brace)	Failure probability## P_{fq}
A-1	$15^{\$} \rightarrow 17 \rightarrow 10 \rightarrow 9 \rightarrow 12 \rightarrow 11$	
A-2	$15^{\$} \rightarrow 17 \rightarrow 9 \rightarrow 12 \rightarrow 10 \rightarrow 20 \rightarrow 8$	0.2132×10^{-3}
A-3	others [13]*	
B-1	$15 \rightarrow 17 \rightarrow 10 \rightarrow 9 \rightarrow 12 \rightarrow 20 \rightarrow 7 \rightarrow 23^{\$} \rightarrow 11$	
B-2	$15 \rightarrow 17 \rightarrow 9 \rightarrow 12 \rightarrow 10 \rightarrow 20 \rightarrow 7 \rightarrow 23^{\$} \rightarrow 8$	0.6740×10^{-5}
B-3	others [4]	
C-1	$17^{\$} \rightarrow 15 \rightarrow 10 \rightarrow 9 \rightarrow 12 \rightarrow 11$	
C-2	$17^{\$} \rightarrow 15 \rightarrow 9 \rightarrow 12 \rightarrow 11 \rightarrow 13$	0.4085×10^{-6}
C-3	others [9]	
D-1	$21^{\$} \rightarrow 23 \cdot (5,6,7,8)^{**}$	0.1881×10^{-4}
Computation time (sec)		95.2

Collapse type



\# Criterion of structural failure is based on singularity of reduced total structure stiffness matrix.

* The figure in brackets designates the number of selected failure paths.

** The figures in parenthesis designate the element end with $\overline{Z}_i^{(p)} \leq 0$.

$ The element end whose failure probability is the smallest.

\## $P_{fq} = \min_{p \in \{1,2,\ldots,p_q\}} P\,[\,Z_{r_p(q)}^{(p)} \leq 0\,]$

Heuristic parameter ; $(a_1, a_2, a_3) = (1.1, 0.0, 2)$

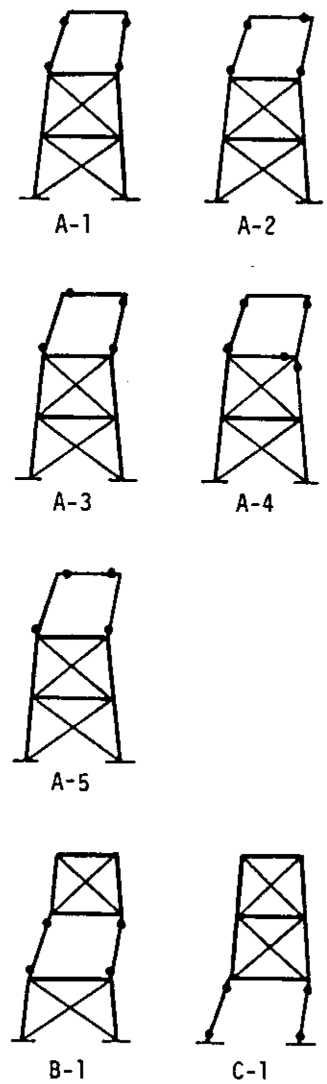
Initial reference value $P_{fpM} = 0.1825 \times 10^{-3}$; $15 \rightarrow 17 \rightarrow 10 \rightarrow 9 \rightarrow 12 \rightarrow 11$

Table III Calculated results of the jacket-type structure with ductile braces

$\gamma_1 = \gamma_2 = \gamma = 3.0,\ \varepsilon^{\#} = 0.001,\ CV_{R_i}/CV_{L_j} = 0.15/0.30$

Type	Failure mode (Ductile brace)		Failure probability## P_{fq}
A-1	(15,17,10,9,12,11)	[5]*	0.7861×10^{-8}
A-2	(15,17,10,9,11,14)	[4]	0.4751×10^{-9}
A-3	(15,17,9,12,11,13)	[2]	0.1879×10^{-9}
A-4	(15,17,10,9,12,20,8)	[1]	0.3030×10^{-10}
A-5	(15,17,9,11,14,13)	[2]	0.2564×10^{-10}
B-1	(21,23,5,6,7,8)	[15]	0.2347×10^{-8}
C-1	(27,29,3,1,4,2)	[1]	0.1214×10^{-10}
Computation time (sec)			660.6

Collapse type (Collapse mode)



\# Criterion of structural failure is based on singularity of reduced total structure stiffness matrix.

* The figure in brackets designates the number of selected failure paths.

\## $$P_{fq} = \min_{p\in\{1,2,\ldots,p_q\}} P\left[\, Z_{r_p(q)}^{(p)} \leq 0 \,\right]$$

$$= P\left[\, Z_{r_p(q)}^{(p_q)} \leq 0 \,\right]$$

Heuristic parameter ; $(a_1, a_2, a_3) = (1.1, 0.0, 2)$

Initial reference value

$P_{fpM} = 0.6017\times10^{-8}$; 15→17→10→9→12→11

for the case of brittle and ductile braces, respectively.

It is seen that the dominant failure modes of both cases are essentially similar, which is formed by failure of the columns and braces in the top story. However, the probabilities of occurrence for the case of brittle braces are very large. Moreover, it is seen from Table II that element end 15 is a critical brittle one whose failure triggers a chain-reaction failure resulting in a total collapse.

Transverse Ring of Ships

Fig. 4 shows a plane frame structure which is modelled for a transverse ring of a medium size tanker under four notional load conditions. The probabilistic analysis of plastic collapse is carried out for the numerical data of the structure given in Fig.4, Tables IV and V. The applied loads are estimated, based on the load condition for the direct calculation suggested by the Japan Classification Society of Ships (NK), and the lengths of rigid bodies are estimated, using "the span point for bending" given in Ref.(22). It is assumed that the strengths of the nodes and the applied loads are mutually independent normal random variables.

The results for the loading condition "case 1-1" are listed in Table VI. The first column indicates the selected failure paths. In the second column, probabilities of occurrence of the failure paths are given when the combined effect of bending moment, shearing force and axial force(a=1, b=0.5, c=1)is considered. The numbers in brackets are those of the selected failure paths. The third column shows those corresponding to the case where the combined effect of bending moment and axial force is considered (a=1, b=0, c=1). The fourth column shows those corresdponding to the case where only bending moment effect is considered(a=b=0,c=1). Further, collapse modes are given in the fifth column. In each column the number in parentheses indicates the central safety factor corresponding to each failure path:

$$\zeta_{SF} = (\bar{R}_i + \sum_{k=1}^{pq-1} a_{ir_k}^{(pq)} \bar{R}_{r_k}) / \sum_{j=1}^{3l} b_{ij}^{(pq)} \bar{L}_j \qquad (32)$$

It is seen from the table that the dominant failure mode of each case is essentially similar, which is formed by failure of wing tank. However, the probabilities of occurrence with combined effect considered are very large, as seen in the failure paths A-1 and A-3 of the table. Moreover, it is seen from comparison between the safety factor and failure probabilities having the same failure path, that the deterministically dominant path is not always stochastically relevant.

Finally, Table VII shows the most dominant collapse mode based on probabilistic analysis for the transverse ring of the tanker under some notional load conditions. It is seen from this table that full load condition with empty centre tank "case 1-1" is the severest. In Table VII, the dominant failure modes are also given for two other types of ships, i.e., a tanker(DW 240,000t) and an ore carrier(DW 50,000t), which configurations and numerical data are shown in Fig. 5 and Tables VIII, and Fig. 6 and Table IX, respectively.

CONCLUDING REMARKS

The methods are presented for the reliability assessment of marine structures, which are modeled as relatively stiff frames and subjected to quasi-static extreme loads, based on the ultimate collapse analysis. The methods are applied to the offshore jacket platform with brittle members and the transverse ring of ships under some notional load conditions. For the former example, the effect of the brittle member on probabilistic collapse of the jacket platform is discussed. For the lattter example, effects of combined loads and notional load conditions on probabilistic properties of the plastic collapse are discussed.

Although this paper is concerned with the case where the structural system is idealized as plane frame structures, it is possible for this method to be extended to reliability analysis of spatial frame structures by incorporating the terms of biaxial bending moment, torsional moment, etc. in the plasticity condition of the equation (1).

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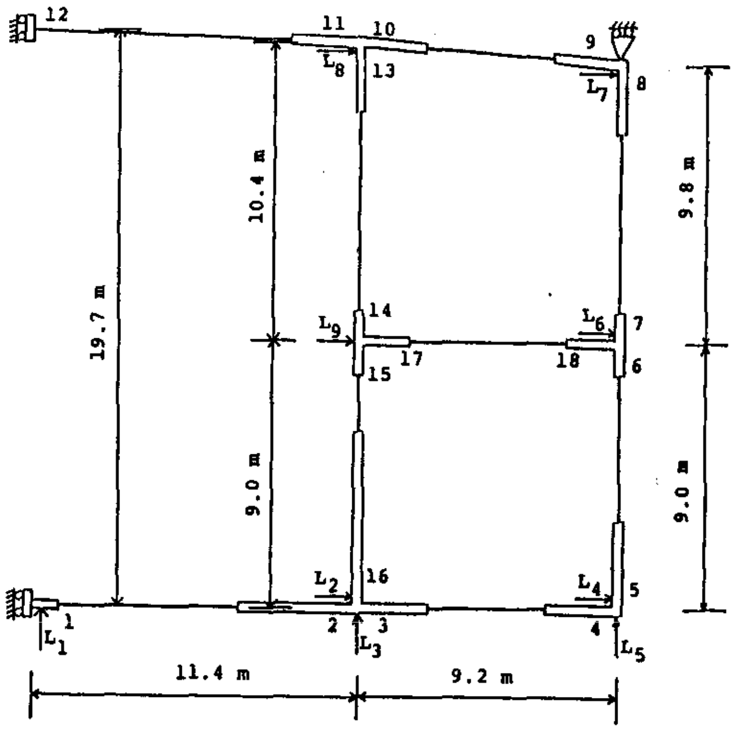


Fig. 4 Transverse ring of tanker 1 (DW 60,000t)

Table IV Numerical data of tanker 1

Element end number	*Cross sectional area* $A_i=A_{pi}$ m²	*Cross sectional area of web* $A_{wi}=AF_{pi}$ m²	*Moment of inertia* I_i m⁴	*Mean value of reference strength* $\overline{R}_i=\overline{\sigma}_{Yi}AZ_{pi}$ kNm	*Length of rigid body* s_{1i} s_{2i} m
1, 2	0.126	0.047	0.187	38950.0	1.0 4.2
3, 4	0.114	0.037	0.111	25640.0	2.5 2.5
5, 6	0.088	0.024	0.042	12850.0	3.1 1.1
7, 8	0.088	0.024	0.042	12850.0	1.1 2.4
9, 10	0.100	0.024	0.043	12910.0	2.4 2.3
11, 12	0.100	0.025	0.044	12700.0	2.4 0.0
13, 14	0.078	0.026	0.040	13490.0	2.4 1.1
15, 16	0.088	0.026	0.043	13540.0	1.1 6.0
17, 18	0.033	0.019	0.013	6730.0	1.8 1.9

Young's modulus E = 210 GPa
Mean value of yield stress $\overline{\sigma}_{Yi}$ = 353 MPa
Coeff. of variation of yield stress $CV_{\sigma_{Yi}}$ = 0.05

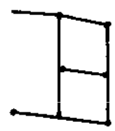
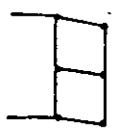
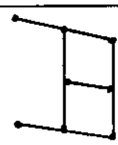
Table V Notional load conditions for the transverse ring of **tanker 1**

Load no.	*Case 1-1*	*Case 1-2*	*Case 1-3*	*Case 1-4*
	Full loaded conditons		*Ballast conditions*	
	Empty centre tank	*Empty wing tanks*	*Empty centre tank*	*Empty wing tanks*
$\overline{L}_1$	2800.0*	-1070.0	1610.0	-2270.0
$\overline{L}_2$	-2970.0	3070.0	-2970.0	3070.0
$\overline{L}_3$	1910.0	1520.0	-390.0	-780.0
$\overline{L}_4$	820.0	-2150.0	1570.0	-1400.0
$\overline{L}_5$	-884.0	2600.0	-1940.0	-1490.0
$\overline{L}_6$	590.0	-2980.0	2120.0	-1450.0
$\overline{L}_7$	-1010.0	-550.0	450.0	0.0
$\overline{L}_8$	-450.0	500.0	-450.0	500.0
$\overline{L}_9$	-3570.0	3750.0	-3570.0	3750.0

* These values denote the mean values of loads.
Coefficients of variation of loads $CV_{L_j}=0.30$ $(j=1,2,\cdots,9)$.

Table VI Failure modes and their probabilities of occurrence for the transverse ring of **tanker 1** in the notional load condition "case 1-1"

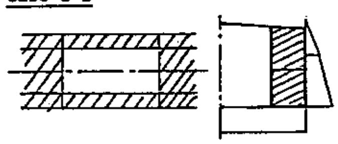
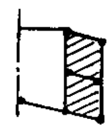
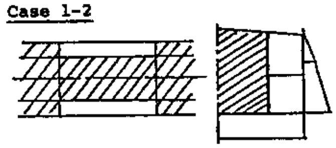
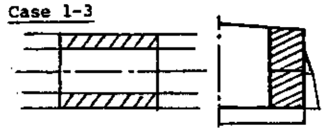
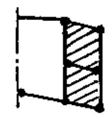
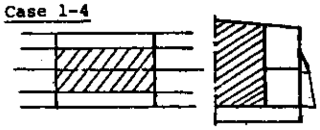
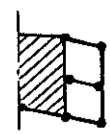
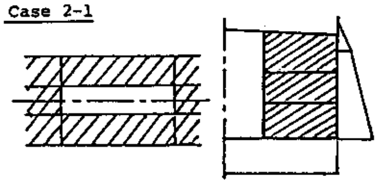
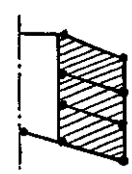
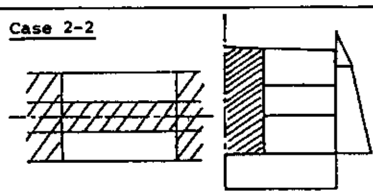
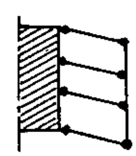
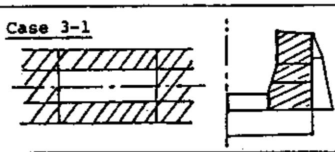
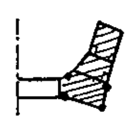
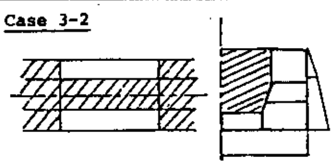
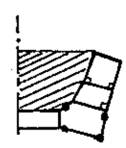
$\gamma_1 = \gamma_2 = \gamma = 4.0$, $\epsilon=0.001$, $CV_{Rj}/CV_{Lj}=0.05/0.3$

No.	*Failure paths*	*Failure probability* P_{fq}: *Bending moment, axial force and shearing force interaction considered* (a=1,b=0.5)	*Bending moment and axial force interaction considered* (a=1,b=0)	*Bending moment only* (a=0,b=0)	*Collapse mode*
A-1.	(17,1,8,18,10,5,16,	7) 0.3495×10^{-2} [38] (1.710)*	7,13) 0.1193×10^{-6} [68] (2.435)	7) 0.1250×10^{-9} [118] (2.938)	
-2.		15) 0.3463×10^{-2} [75] (1.614)	7) 0.7958×10^{-7} [19] (2.454)	—	
-3.	(16,17,18,1,9,10,5,	15) 0.2760×10^{-2} [8] (1.645)	7) 0.1282×10^{-7} [42] (2.564)	7,13) 0.8329×10^{-11} [7] (3.067)	
-4.		7) 0.2617×10^{-2} [38] (1.736)	13,15) 0.1376×10^{-10} [5] (2.582)	—	
-5.	(17,18,1,8,18,10,4,	7) 0.1395×10^{-2} [22] (1.798)	—	7) 0.4844×10^{-11} [39] (3.170)	
-6.	others	$< 0.11\times10^{-2}$ [107]	$< 0.25\times10^{-7}$ [99]	$< 0.85\times10^{-11}$ [90]	
B-1.	(17,18,8,5,10,3,	15,16) 0.6399×10^{-6} [3] (2.001)	—	1,16) 0.1001×10^{-16} [6] (2.996)	
-2.		7) 0.5683×10^{-6} [1] (2.090)	—	—	
-3.	(18,16,17,9,10,5,3,	—	—	1,13) 0.1465×10^{-14} [2] (3.090)	
-4.	(17,16,18,8,13,5,11,3,	—	—	1) 0.2184×10^{-16} [19] (3.019)	
C-1.	(18,16,17,8,1,13,5,12,	—	—	7) 0.4884×10^{-11} [13] (2.986)	
	Total number of selected paths	[292]	[233]	[294]	
	Computation time (sec)	102.2	88.2	85.6	

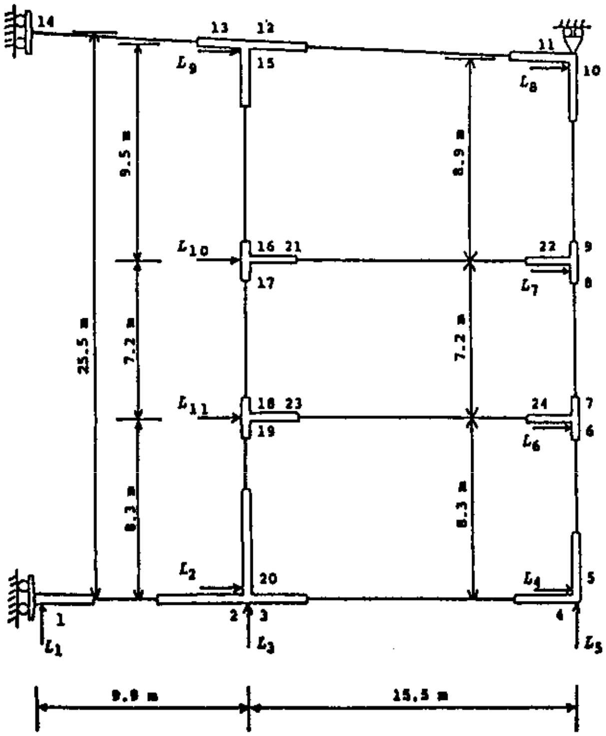
* central safety factor $= \left(\overline{R}_i + \sum_{k=1}^{p_q-1} a_{ir_k}^{(pq)} \overline{R}_{r_k}\right) \Big/ \sum_{j=1}^{3\ell} b_{ij}^{(pq)} \overline{L}_j$

Table VII The most dominant collapse mode for the transverse structure of various ships under some notional load conditions

$CV_{\sigma_{Yi}}/CV_{L_j}$=0.05/0.3

Type of ships	*Type of structures and notional load conditions*		*The most dominant collapse mode and its probability of occurrence*		
			Coeff. of axial and shearing force effect	*Failure probability*	*The most dominant collapse mode*
Tanker 1 DW 60,000 t	*Full loaded conditions*	Case 1-1 	a=1, b=0.5 a=1, b=0 a=0, b=0	0.3495 × 10^{-2} (1.71)* 0.1193 × 10^{-5} (2.44) 0.1250 × 10^{-9} (2.94)	
		Case 1-2 	a=1, b=0.5 a=1, b=0 a=0, b=0	< 0.1 × 10^{-18}	———
	Ballast conditions	Case 1-3 	a=1, b=0.5 a=1, b=0 a=0, b=0	0.2808 × 10^{-16} < 0.1 × 10^{-18}	
		Case 1-4 	a=1, b=0.5 a=1, b=0 a=0 b=0	0.1057 × 10^{-5} 0.1325 × 10^{-14} < 0.1 × 10^{-18}	
Tanker 2 DW 240,000 t	*Full loaded conditions*	Case 2-1 	a=1, b=0.5 a=1, b=0 a=0, b=0	0.3896 × 10^{-4} 0.1640 × 10^{-10} 0.5120 × 10^{-13}	
		Case 2-2 	a=1, b=0.5 a=1, b=0 a=0, b=0	0.7482 × 10^{-2} (1.68) 0.3625 × 10^{-5} (2.27) 0.7446 × 10^{-8} (2.95)	
Ore carrier DW 50,000 t	*Full loaded conditions*	Case 3-1 	a=1, b=0.5 a=1, b=0 a=0, b=0	0.3663 × 10^{-11} 0.2226 × 10^{-16} < 0.1 × 10^{-18}	
		Case 3-2 	a=1, b=0.5 a=1, b=0 a=0, b=0	0.2066 × 10^{-2} (3.26) 0.1519 × 10^{-3} (4.12) 0.1495 × 10^{-5} (5.75)	

* The value in parentheses indicates the safety factor given in Eq. (32).



Case 2-1	Case 2-2
$\bar{L}_1$ = 2920 kN	$\bar{L}_1$ = -474 kN
$\bar{L}_2$ = -3160 kN	$\bar{L}_2$ = 3160 kN
$\bar{L}_3$ = 2020 kN	$\bar{L}_3$ = 5120 kN
$\bar{L}_4$ = 385 kN	$\bar{L}_4$ = -2780 kN
$\bar{L}_5$ = -905 kN	$\bar{L}_5$ = 5600 kN
$\bar{L}_6$ = 230 kN	$\bar{L}_6$ = -3870 kN
$\bar{L}_7$ = -370 kN	$\bar{L}_7$ = -2500 kN
$\bar{L}_8$ = -420 kN	$\bar{L}_8$ = -741 kN
$\bar{L}_9$ = -318 kN	$\bar{L}_9$ = 318 kN
$\bar{L}_{10}$ = -2130 kN	$\bar{L}_{10}$ = 2130 kN
$\bar{L}_{11}$ = -4100 kN	$\bar{L}_{11}$ = 4100 kN
CV_{Lj} = 0.3	CV_{Lj} = 0.3
(j=1,2,···,12)	(j=1,2,···,12)

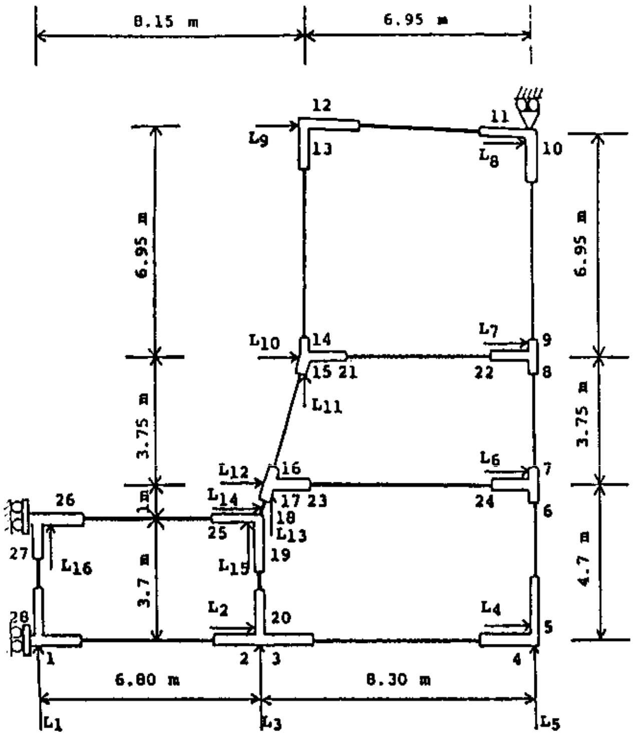
Case 2-1 : Wing tank fully loaded
Case 2-2 : Main tank fully loaded

Fig. 5 Transverse ring of tanker 2 (DW 240,000t)

Table VIII Numerical data of tanker 2

Element end number	*Cross sectional area* A_i=A_{pi} m²	*Cross sectional area of web* A_{wi}=AF_{pi} m²	*Moment of inertia* I_i m⁴	*Mean value of reference strength* $\bar{R}_i$ kNm	*Length of rigid body* s_{1i} s_{2i} m
1, 2	0.183	0.064	0.438	59200.0	2.8 4.2
3, 4	0.176	0.054	0.255	43100.0	2.8 2.9
5, 6	0.135	0.045	0.156	29400.0	3.1 1.0
7, 8	0.135	0.045	0.156	29400.0	1.0 1.0
9, 10	0.126	0.035	0.147	25900.0	1.0 3.0
11, 12	0.136	0.030	0.076	14600.0	3.0 2.9
13, 14	0.137	0.030	0.079	15100.0	2.2 0.0
15, 16	0.089	0.035	0.117	24800.0	3.0 0.9
17, 18	0.099	0.035	0.127	25700.0	0.9 1.0
19, 20	0.116	0.045	0.142	29200.0	1.0 5.1
21, 22	0.043	0.018	0.016	6650.0	2.3 2.3
23, 24	0.056	0.025	0.026	9700.0	2.3 2.3

Young's modulus E = 210 GPa
Mean value of yield stress $\bar{\sigma}_{Yi}$ = 276 MPa



Case 3-1	Case 3-2
$\bar{L}_1$ = 1430 kN	$\bar{L}_1$ = 1430 kN
$\bar{L}_2$ = -995 kN	$\bar{L}_2$ = 0 kN
$\bar{L}_3$ = 1040 kN	$\bar{L}_3$ = 3390 kN
$\bar{L}_4$ = 118 kN	$\bar{L}_4$ = -1050 kN
$\bar{L}_5$ = -350 kN	$\bar{L}_5$ = 1960 kN
$\bar{L}_6$ = -101 kN	$\bar{L}_6$ = -1450 kN
$\bar{L}_7$ = -474 kN	$\bar{L}_7$ = -1400 kN
$\bar{L}_8$ = -570 kN	$\bar{L}_8$ = -764 kN
$\bar{L}_9$ = -260 kN	$\bar{L}_9$ = 260 kN
$\bar{L}_{10}$ = -944 kN	$\bar{L}_{10}$ = 944 kN
$\bar{L}_{11}$ = 148 kN	$\bar{L}_{11}$ = -148 kN
$\bar{L}_{12}$ = -529 kN	$\bar{L}_{12}$ = 529 kN
$\bar{L}_{13}$ = 134 kN	$\bar{L}_{13}$ = -134 kN
$\bar{L}_{14}$ = -917 kN	$\bar{L}_{14}$ = 133 kN
$\bar{L}_{15}$ = 49 kN	$\bar{L}_{15}$ = -1150 kN
$\bar{L}_{16}$ = 0 kN	$\bar{L}_{16}$ = -1100 kN
CV_{Lj} = 0.3 $(j=1,2,\cdots,16)$	CV_{Lj} = 0.3 $(j=1,2,\cdots,16)$

Case 3-1 : Wing tank fully loaded
Case 3-2 : Main tank fully loaded

Fig. 6 Transverse structure of an ore carrier(DW 50,000t)

Table IX Numerical data of the ore carrier

Element end number	*Cross sectional area* $A_i=A_{pi}$ m²	*Cross sectional area of web* $A_{wi}=AF_{pi}$ m²	*Moment of inertia* I_i m⁴	*Mean value of reference strength* $\bar{R}_i$ kNm	*Length of rigid body* s_{1i}, s_{2i} m
1, 2	0.133	0.044	0.045	13500.0	1.3 1.4
3, 4	0.133	0.044	0.045	13500.0	1.7 1.7
5, 6	0.158	0.046	0.035	12300.0	2.0 0.5
7, 8	0.158	0.046	0.035	12300.0	0.5 0.5
9, 10	0.158	0.046	0.035	12300.0	0.5 1.4
11, 12	0.166	0.044	0.027	10500.0	1.6 1.6
13, 14	0.055	0.017	0.013	4400.0	1.4 0.5
15, 16	0.080	0.024	0.019	6400.0	0.5 0.5
17, 18	0.080	0.024	0.019	6400.0	0.5 0.0
19, 20	0.130	0.047	0.031	10600.0	1.6 1.6
21, 22	0.046	0.015	0.059	4300.0	1.3 1.3
23, 24	0.050	0.017	0.066	4700.0	1.3 1.3
25, 26	0.098	0.024	0.013	5350.0	1.4 1.4
27, 28	0.041	0.011	0.025	1750.0	1.2 1.6

Young's modulus E = 210 GPa
Mean value of yield stress $\bar{\sigma}_{Yi}$ = 276 MPa

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