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Extreme Value Distributions of Wave Loads and Their Application to Marine Structures

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INTRODUCTION

For the purpose of determining the random wave loads acting on offshore structures and ocean-going vessels, it is often assumed that ocean waves can be modeled as a stationary Gaussian process with zero mean. The process can be of any band-width, although the conservative assumption of a narrow-band spectral density is usually made for simplifying the analysis. Based on this probabilistic model, the extreme value distribution of the wave amplitude or the wave load acting on the structure is determined using one of several available methods. The extreme value distribution is then, either used to estimate a design load associated with a prescribed probability of exceedence or alternatively, used in reliability analysis in conjunction with the strength characteristics of the structure [1].

The purpose of this study is to compare several extreme load distributions and to determine their impact on the probabilities of exceedence. The extreme value distribution of peaks of a stationary Gaussian process of any-band width representing the load on a marine structure was determined on the basis of four different methods. In the first, the peaks were assumed to be statistically independent and indentically distributed, and the extreme value distribution of the largest in N-peaks was determined using classical order statistics. In the second, a discrete point process was assumed in order to determine the asymptotic type-I distribution based on Rice's [2] initial distribution. Cramer's procedure [3,4] was used for determining the resulting asymptotic distribution. Conventional up-crossing analysis was used in the third method for determining the extreme value distribution. Finally a two-stage description of the random process which leads to an extreme distribution derived by Vanmarcke [6] was the basis for the fourth method.

The four resulting extreme distributions were then compared numerically for the case of a relatively narrow-band process and typical values of load parameters determined for an ocean-going vessel.

EXTREME VALUE DISTRIBUTIONS ASSOCIATED WITH A STATIONARY GAUSSIAN PROCESS OF ANY BAND-WIDTH

The four methods mentioned in the Introduction to determine the extreme value distribution of wave loads acting on a marine structure will be discussed in this section. In all cases a general stationary Gaussian process of any band width will be considered to represent the wave load. In the following sections, special cases of practical interest will be addressed.

Distribution of the largest peak in a sequence of N peaks using order statistics

The distribution of the largest peak in a sequence of N peaks can be determined using standard order statistics. Consider a sequence of random variables $z_1, z_2, \ldots z_N$ representing the peaks of a load on a marine structure. Assuming that these peaks are identically distributed and statistically independent, the cumulative distribution function (cdf) of the largest one using order statistics is given by:

$$F_{Z_N}(z) = P\left[\max(z_1, z_2, \ldots, z_N) \le z\right] = \left[F_Z(z,\varepsilon)\right]^N \qquad (1)$$

Where $F_Z(z,\varepsilon)$ is the cumulative distribution function of the load peaks (maxima) and ε is the spectral width parameter defined as:

$$\varepsilon^2 = 1 - \frac{m_2^2}{m_0 m_4}$$

$$m_n = \int_{-\infty}^{+\infty} \omega^n S(\omega)\, d\omega \quad , \quad n = 0, 2, 4 \qquad (2)$$

The probability density function (pdf) of the largest peak is determined by differentiating equation (1) with respect to z, thus:

$$f_{Z_N}(z) = N\left[F_Z(z,\varepsilon)\right]^{N-1} \cdot f_Z(z,\varepsilon) \qquad (3)$$

where $f_Z(z, \varepsilon)$ is the pdf of the load peaks (see equation (19)).

For linear systems subjected to ocean waves represented by a Gaussian process, the loads can be also modeled by a Gaussian process (see for example reference [1]). The peaks of the Gaussian load process, in general, follow Rice's distribution given by [2]:

$$F_Z(z,\varepsilon) = \Phi\left(\frac{z-m_S}{\varepsilon\sqrt{m_0}}\right) - \sqrt{1-\varepsilon^2}\cdot e^{-\frac{1}{2}\left(\frac{z-m_S}{\sqrt{m_0}}\right)^2} \cdot \Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\cdot\frac{z-m_S}{\sqrt{m_0}}\right) \tag{4}$$

where

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx$$

and m_S is the mean value of the process.

The spectral width parameter ε is defined by equation (2). It should be noted that the random variable z represents the load peak height above or below the mean value m_S, that is, equation (4) includes both positive and negative maxima.

For a narrow band spectrum $S(\omega)$, the band width ε approaches zero and equation (4) reduces to the Rayleigh distribution function. Similarly, for a wide band spectrum, ε approaches one and equation (4) reduces to the standard Gaussian distribution function, i.e., the distribution of the load peak reduces to the distribution of the load deviation from the mean value.

Equations (1) and (3) give the cdf and the pdf of the extreme peak in N peaks, respectively. Equation (4) is to be used in these two equations as the initial distribution for a load random process of any band width ε and mean value m_S.

<u>Asymptotic type I distribution</u>

It is known that as the number of peaks N increases without bound a limiting or asymptotic form of the extreme value distribution (equations (1) and (3)) is reached. The asymptotic form of an extreme value distribution does not depend, in general, on the exact form of the initial distribution; it depends only on the tail behavior of the initial distribution. The parameters of the asymptotic distribution depend however on the exact form of the initial distribution [3].

Cramer's method developed in reference [4] and summarized in [3] can be used to derive asymptotic distributions in general. In our case, Rice's distributions given by equation (4) will be used as the initial distribution. Following Cramer's method a new random variable may be defined as:

$$\zeta_N = N\left[1 - F_Z(Z_n,\varepsilon)\right]$$

where

$$F_Z(Z_n,\varepsilon) = \Phi\left(\frac{Z_n-m_s}{\varepsilon\sqrt{m_0}}\right) - \sqrt{1-\varepsilon^2}\cdot e^{-\frac{1}{2}\left(\frac{Z_n-m_s}{\sqrt{m_0}}\right)^2} \cdot \Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\cdot\frac{Z_n-m_s}{\sqrt{m_0}}\right) \tag{5}$$

therefore,

$$\zeta_N = N\left[\Phi\left(\frac{m_s-Z_n}{\varepsilon\sqrt{m_0}}\right) + \sqrt{1-\varepsilon^2}\cdot e^{-\frac{1}{2}\left(\frac{Z_n-m_0}{\sqrt{m_0}}\right)^2} \cdot \Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\cdot\frac{Z_n-m_s}{\sqrt{m_0}}\right)\right] \tag{6}$$

As N becomes large, Cramer has shown that the asymptotic distribution of Z_N is obtained from:

$$F_{Z_N}(z,\varepsilon) = e^{-g(z,\varepsilon)} \tag{7}$$

and

$$f_{Z_n}(z,\varepsilon) = -\frac{dg(z,\varepsilon)}{dz}\,e^{-g(z,\varepsilon)} \tag{8}$$

where $g(z, \varepsilon)$ is the right side of equation (6). Therefore, for large N the asymptotic distribution function from (6) and (7) is:

$$F_{Z_N}(z,\varepsilon) = \exp\left\{-N\left[\Phi\left(\frac{m_S-z}{\varepsilon\sqrt{m_0}}\right) + \sqrt{1-\varepsilon^2}\; e^{-\frac{1}{2}\left(\frac{z-m_S}{\sqrt{m_0}}\right)^2} \cdot \Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\cdot\frac{z-m_S}{\sqrt{m_0}}\right)\right]\right\} \tag{9}$$

that is, the asymptotic form is double exponential and the cumulative distribution itself depends on N.

Several years after the appearance of Cramer's book Gumbel [5] classified the asymptotic distribution of extremes in three types: (type I) a double exponential form, (type II) an expontial form, and (type III) an exponential form with an upper bound. Convergence of an initial distribution to one of the three types depends largely on the tail behavior of the initial distribution. An initial distribution with an exponentially decaying tail in direction of the extreme will converge to type I asymptotic distribution, i.e., the double exponential form. Equation (9) exhibits this behavior.

Gumbel's analysis and classification provide another method for deriving the asymptotic distribution and may be in a form easier to handle than that given by equation (9). The cdf of type I asymptotic form as given by Gumbel is:

$$F_{Z_N}(z) = \exp\left[- e^{-\alpha_N (z - u_N)}\right] \tag{10}$$

where u_N is the characteristic largest value of the initial variate Z and α_N is an inverse measure of dispersion of Z_N. These parameters, u_N and α_N, have to be determined and depend on the form of the initial distribution.

The corresponding pdf is given by:

$$f_{Z_N}(z) = \alpha_N e^{-\alpha_N (z - u_N)} \cdot \exp\left[- e^{-\alpha_N (z - u_N)}\right] \tag{11}$$

The mean and standard deviation of the extreme value Z_N are given, respectively, by:

$$\mu_{Z_n} = u_N + \frac{0.5772}{\alpha_N} \tag{12}$$

$$\sigma_{Z_n} = \frac{\pi}{\sqrt{6}\,\alpha_N} \tag{13}$$

The parameters u_N and α_N will now be determined for Rice's distribution given by equation (4) as an initial distribution.

The characteristic largest value u_N is defined as the particular value of the random load Z such that in a sample of size N the expected number of sample values larger than u_N is one, i.e.,

$$N\left[1 - F_Z(u_N)\right] = 1.0$$

or

$$F_Z(u_N) = 1 - \frac{1}{N} \tag{14}$$

Using equation (4) for the initial distribution $F_Z(.)$, equation (14) becomes:

$$\Phi\left(\frac{u_N - m_s}{\varepsilon\sqrt{m_0}}\right) - \sqrt{1-\varepsilon^2}\; e^{-\frac{1}{2}\left(\frac{u_N - m_s}{\sqrt{m_0}}\right)^2} \cdot \Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon} \cdot \frac{u_N - m_s}{\sqrt{m_0}}\right) = 1 - \frac{1}{N} \tag{15}$$

Equation (15) is then solved for u_N and yields

$$u_N = m_S \pm \left\{2\, m_0 \ln\left[\frac{\sqrt{1-\varepsilon^2}\;\Phi(\beta)}{\frac{1}{N} - \Phi(-\alpha)}\right]\right\}^{\frac{1}{2}} \tag{16}$$

where

$$\alpha = \frac{u_N - m_S}{\varepsilon\sqrt{m_0}}$$

and

$$\beta = \sqrt{1-\varepsilon^2} \cdot \alpha \tag{17}$$

The plus sign in equation (16) should be used if the mean value m_S is positive in order to obtain the larger characteristic value. It should be noted that both α and β contain u_N as defined in (17); therefore, an iterative procedure must be used for determining u_N. To start the iterative procedure an initial value for u_N is necessary and may be taken as $u_N = m_S + \sqrt{2 m_0 \ln N}$. The corresponding values of α, β, $\Phi(-\alpha)$ and $\Phi(\beta)$ can then be determined. Equation (16) is then checked to see if the right side is equal to the left side, otherwise a new value of u_N equals the right side of equation (16) should be used in the second step of the iterative procedure. Three or four steps are usually sufficient for convergence.

Based on Gumbel's analysis, the second parameter α_N can be determined from:

$$\alpha_N = N f_Z(u_N) \tag{18}$$

where $f_Z(u_N)$ is the value of the pdf of the initial distribution of the load Z at the characteristic largest value u_N.

The pdf of the initial distribution (Rice) is obtained by taking the derivative of equation (4) with respect to z and is given by:

$$f_Z(z,\varepsilon) = \frac{\varepsilon}{\sqrt{2\pi m_0}} e^{-\frac{1}{2}\left(\frac{z-m_s}{\varepsilon\sqrt{m_0}}\right)^2} + \frac{\sqrt{1-\varepsilon^2}}{\sqrt{m_0}} \cdot \left(\frac{z-m_s}{\sqrt{m_0}}\right) e^{-\frac{1}{2}\left(\frac{z-m_s}{\sqrt{m_0}}\right)^2} \cdot \Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon} \cdot \frac{z-m_s}{\sqrt{m_0}}\right) \tag{19}$$

Using equation (19) in (18), the parameter α_N can be written in the form:

$$\alpha_N = \frac{N\varepsilon}{\sqrt{2\pi m_0}} e^{-\frac{\alpha^2}{2}} + \frac{N\varepsilon\beta}{\sqrt{m_0}} e^{-\frac{\alpha^2}{2}\varepsilon^2} \cdot \Phi(\beta) \tag{20}$$

where α and β are defined earlier by equations (17).

It should be noted that u_N and α_N as given by equations (16) and (20), respectively, completely define Gumbel's asymptotic type I distribution given by equation (10) with Rice's distribution as an initial distribution.

<u>Extreme value distribution based on upcrossing analysis</u>

The distribution of the largest peak can be determined from upcrossing analysis of a time history of a stationary random process instead of the peak analysis presented above. For example, the number of N peaks can be changed to a time interval T in the upcrossing analysis and the problem of determining the characteristics of the largest peak in N peaks becomes that of evaluating the characteristics of the maximum crest of a stationary ergodic Gaussian random process X(t) during a period T. The assumption of the statistical independence of the peaks is usually replaced by the assumption that upcrossing of a level x by X(t) are statistically independent. This leads to the Poisson's upcrossing process which is true only in the asymptotic sense (as $x \to \infty$; $T \to \infty$).

From upcrossing analysis it can be shown (see for example [1]) that the probability that the largest value exceeds a certain level x during a period T is given by:

$$P\left[\max\left(X(t) \; ; \; 0 \le t \le x\right) \le x\right] = e^{-\nu_x^+ T} \tag{21}$$

where

$$\nu_x^+ = \nu_0 \, e^{-\frac{1}{2}\left(\frac{x-m_s}{\sqrt{m_0}}\right)^2} \tag{22}$$

and

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}} \quad 1/\text{sec} \tag{23}$$

Therefore the cdf of the largest X is

$$F_X(x) = \exp\left[-\nu_0 T \, e^{-\frac{1}{2}\left(\frac{x-m_s}{\sqrt{m_0}}\right)^2}\right] \tag{24}$$

that is, it has a double exponential form although quite different from equation (10) with u_N and α_N given by (16) and (20).

<u>Extreme value distribution based on a two-state description of a random process</u>

Vanmarcke [6] estimated the probability distribution of the time to first passage across a specified barrier for a Gaussian stationary random process. In his analysis he considered a two-state description of the time history X(t) relative to the specified barrier. Based on his results the distribution of the extreme value may be determined from:

$$F_X(x) = \exp\left[-\nu_0 T \left(\frac{1 - e^{-\sqrt{2\pi}\, q\left(\frac{x-m_s}{\sqrt{m_0}}\right)}}{1 - e^{-\frac{1}{2}\left(\frac{x-m_s}{\sqrt{m_0}}\right)^2}}\right) \cdot e^{-\frac{1}{2}\left(\frac{x-m_s}{\sqrt{m_0}}\right)^2}\right] \tag{25}$$

where q is a band width parameter defined as

$$q = \sqrt{1 - \frac{m_1^2}{m_0 m_2}} \qquad 0 \le q \le 1 \tag{26}$$

SPECIAL CASES OF THE EXTREME VALUE DISTRIBUTIONS

The extreme value distributions representing the load on a marine structure discussed above are applicable to stationary Gaussian processes of any band width. These distributions can be simplified if one considers the special cases of a narrow and wide band processes. The former special case is particularly important for practical applications and usually gives a conservative estimate of the calculated probabilities of exceedence. In this section both cases will be briefly discussed.

Narrow-band wave load process

Each of the extreme value distributions is reduced to the special case of a narrow-band Gaussian process in this section. Starting with the extreme value distribution based on the largest peak in a sequence of N-peaks, we notice that for the narrow-band case $\varepsilon \to 0$. Equations (1) and (3) defining the form of the extreme distribution remain unaltered. However, equations (4) and (19) which define the initial (Rice) distribution reduce to Rayleigh distribution upon substituting $\varepsilon = 0$, i.e.,

$$F_Z(z,0) = 1 - e^{-\frac{1}{2}\left(\frac{z-m_s}{\sqrt{m_0}}\right)^2} \tag{27}$$

and

$$f_Z(z,0) = \frac{z - m_s}{m_0} \cdot e^{-\frac{1}{2}\left(\frac{z-m_s}{\sqrt{m_0}}\right)^2} \tag{28}$$

For the case of asymptotic type I distribution, the narrow-band case can be also developed by inserting $\varepsilon = 0$. Cramer's method for determining the extreme value distribution thus yields (equation (9) with $\varepsilon = 0$):

$$F_{Z_N}(z,0) = \exp\left\{-N\left[1 + e^{-\frac{1}{2}\left(\frac{z-m_s}{\sqrt{m_0}}\right)^2}\right]\right\} \tag{29}$$

Gumbel's general asymptotic distribution given by equations (10) and (11) remain unaltered. The parameter u_N and α_N given by equations (16) and (20), respectively, reduce to:

$$u_N = m_s + \sqrt{2 m_0 \ln N} \tag{30}$$

and

$$\alpha_N = \sqrt{\frac{2 \ln N}{m_0}} \tag{31}$$

Equations (30) and (31) for u_N and α_N are identical to those published in the literature, e.g., in reference [3].

The extreme distribution based on upcrossing analysis given by equation (24) as well as that based on a two-state description of the random process (equation (25)) remain unaltered. In both cases the fact that the process is a narrow-band process is reflected in the number of zero crossing $\nu_0 T$, which is in this case, equal to the number of peaks N. The value of "q" in equation (25) also reflects the band width of the process.

Wide-band wave load process

In this case the band width parameter approaches one. The extreme value distribution based on the largest peak in a sequence of N peaks is thus given by equations (1) and (3) but with an initial distribution obtained by substituting $\varepsilon = 1$ in Rice's equations (4) and (19). These equations reduce to the Gaussian distributions, i.e.,

$$F_Z(z,1) = \frac{1}{\sqrt{2\pi m_0}} \int_{-\infty}^{z} e^{-\frac{1}{2}\left(\frac{z-m_s}{\sqrt{m_0}}\right)^2} dz = \Phi\left(\frac{z-m_s}{\sqrt{m_0}}\right) \tag{32}$$

and

$$f_Z(z,1) = \frac{1}{\sqrt{2\pi m_0}} e^{-\frac{1}{2}\left(\frac{z-m_s}{\sqrt{m_0}}\right)^2} \tag{33}$$

Cramer's method for determing the asymptotic extreme distribution yields for the wide band case ($\varepsilon = 1$ in equation (9)):

$$F_Z(z, 1) = \exp\left[-N\,\Phi\left(\frac{m_s - z}{\sqrt{m_0}}\right)\right] \quad (34)$$

Gumbel's general asymptotic distribution given by equations (10) and (11) remains unaltered. The parameter u_N can be determined from equations (16) or more easily from (15) by substituting $\epsilon = 1$. The parameter α_N is determined from equation (20). These equations reduce to:

$$u_N = m_S - \sqrt{m_0}\,\Phi^{-1}\left(\frac{1}{N}\right) \quad (35)$$

and

$$\alpha_N = \frac{N}{\sqrt{2\pi m_0}}\, e^{-\frac{1}{2}\left(\frac{u_N - m_s}{\sqrt{m_0}}\right)^2} = \frac{N}{\sqrt{2\pi m_0}}\, e^{-\frac{1}{2}\left[\Phi^{-1}\left(\frac{1}{N}\right)\right]^2} \quad (36)$$

Equations (35) and (36) are not identical to equations given by Cramer for these two parameters (see for example [3], page 200). Cramer's solution for these parameters which becomes accurate as $N \to \infty$ is (for a process of mean m_s and variance m_0):

$$u_N = m_S + \sqrt{m_0}\left[\sqrt{2 \ln N} - \frac{\ln \ln N + \ln 4\pi}{2\sqrt{2 \ln N}}\right] \quad (37)$$

and

$$\alpha_N = \sqrt{\frac{2 \ln N}{m_0}} \quad (38)$$

Equations (35) and (37) for u_N were compared numerically for the case of zero mean "m_s" and any variance m_0. Several values of N were considered in the comparison. In all cases the difference between the values of U_N obtained from the two equations was less than one percent. We will consider this to be satisfactory for confirming the U_N equations givn by (35) and (37). If the U_N equation given by Cramer (equation 37) is substituted in the first of equations (36) for α_N, an identical result as that given by equation (38) is obtained. This confirms the validity of equation (36). It should be noted that Cramer's equation (38) for α_N in this case ($\epsilon = 1$) is identical to α_N for a narrow-band process ($\epsilon = 0$) as can be seen by comparing (38) and (31). Comparing the values of U_N for the narrow and wide band cases it is seen that equation (37) differs only from equation (30) because of the presence of a second term in the bracket.

The extreme distributions based on up-crossing analysis given by (24) as well as that based on a two-state description of the random process given by (25) remain unchanged. It should be noted that in both cases νT is generally not equal to the number of peaks N.

COMPARISON OF THE EXTREME VALUE DISTRIBUTIONS

The extreme value distributions of the wave loads discussed above differ from each other in their basic derivation and underlying assumptions. The forms of their equations are drastically different as can be seen by comparing equations (1) and (2); (9); (10) and (11); (24); and (25). It would be interesting now to compare some typical results obtained from the different methods when applied to a marine structure. For this purpose a tanker of length = 763 feet, breadth = 125 feet and depth = 54.5 feet is considered. We will compare the distribution of the extreme wave bending moment acting on the tanker under a storm condition specified by a significant wave height of 29.0 feet and an average wave period of 10.1 seconds. The storm is assumed to be stationary under these conditions for a period of one hour. The following parameters were computed for an earlier application given in [1]:

Still water bending moment (full load)
m_s = 669,037 ft-tons

RMS of wave bending moment
$\sqrt{m_0}$ = 216,450 ft-tons

Average wave moment period = 12.1 seconds

Band width parameter of wave moment spectral density ϵ = 0.337

Number of wave moment peaks in one hour
$$N = \frac{60 \times 60}{12.1} = 297.5$$

The application given in reference [1] showed that if ϵ is assumed to be zero (ideal narrow-band) instead of the 0.337 given above, the resulting error in the expected maximum wave bending moment in N peaks is less than 0.5 percent. This gives an indication that for $\epsilon = 0.337$, it is sufficiently accurate to use the ideal narrow-band equations for our comparison.

Using this assumption and the above values for m_s, $\sqrt{m_0}$ and N, a comparison is made of the cumulative distribution functions of four extreme value distributions. These four distributions are:

Distribution (A): Largest peak in N-peaks as given by equations (1) and (27)

Distribution (B): Asymptotic type I distribution as given by equations (10), (30) and (31)

Distribution (C): Upcrossing analysis as given by equation (24) with $\nu_0 T = N$

Distribution (D): A two-state description as given by equation (25) with $\nu_0 T = N$ and q values = 0.35 and 0.25

The results of the comparison are shown in Table 1 and are plotted on a standard extreme probability paper and on a regular graph paper in figure 1 and 2, respectively.

The probability density function of distribution (A) as given by equations (3), (27) and (28); distribution (B) as given by (11), (30) and (31); and distribution (C) as given by the derivative of equation (24) with respect to x are plotted in figure 3.

DISCUSSION OF THE RESULTS AND CONCLUDING REMARKS

Based on the results given in Table 1 and Figures 1 and 2 one surprising conclusion can be drawn. All extreme value distributions of the wave loads considered produce similar results even though their basic assumptions and derivations differ drastically. In fact, if one inspects the equations representing the cumulative distribution functions of these distributions (equations (1) and (27); (10), (30) and (31); (24); and (25)) one sees that these equations are not similar in form and may conclude erroneously that they would produce very different results.

The extreme distributions based on the largest peak in N peaks (distribution A), upcrossing analysis (distribution B) and a two-state description (distribution C) produce almost identical results as far as the probability of exceedence is concerned as can be seen by inspecting figures 1 and 2. The asymptotic type I distribution (distribution B) results in slightly higher values of probability of exceedence. This is to be expected since the asymptotic distribution is an upper bound extreme distribution and becomes more accurate as the number of load peaks approaches infinity. In the example shown for the tanker, the number of wave bending moment peaks N is approximately 298.

As an example of the difference between the asymptotic distribution and the other distributions, the probability of exceedence of a total bending moment of 2,069,000 ft-ton (including still water bending moment of 669,000 ft-ton) is 0.006 according to the asymptotic distribution (B) and 0.002 according to the other three distributions (A, C and D) as can be seen from Table 1. The insensitivity of the probability of exceedence given by distribution D (two-stage description) to the value of the band width parameter "q" is to be noted.

To summarize the main points in this study, four extreme value distributions of wave loads acting on a marine structure and modeled as a general stationary Gaussian process of any band width have been considered. The peaks of the wave loads thus follow a general distribution given by Rice [2]. The extreme value distributions were then evaluated and, in particular, Gumbel's type I asymptotic distribution parameter was determined using Rice's distribution as an initial distribution. The extreme value distributions were then applied to represent wave loads on a tanker where the spectral band width was determined to be narrow. The results show that Gumbel's asymptotic distribution may be used for conservative upper bound analysis while the other three extreme distributions give slightly lower values. Of these three distributions, the one based on upcrossing analysis (distribution C, equation 24) seems to be the easiest to handle.

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TABLE 1 - COMPARISON OF EXTREME VALUE DISTRIBUTIONS OF WAVE BENDING MOMENT ON A TANKER
CUMULATIVE DISTRIBUTION FUNCTIONS

Total Bending Moment in ft-tons	Distribution A	Distribution B	Distribution C	Distribution D with q = .35	Distribution D with q = .25
1469000	3.554×10^{-3}	1.461×10^{-3}	3.762×10^{-3}	5.480×10^{-3}	9.018×10^{-3}
1519000	0.034	0.026	0.034	0.042	0.056
1569000	0.137	0.132	0.138	0.155	0.180
1619000	0.324	0.324	0.324	0.343	0.372
1669000	0.537	0.534	0.537	0.552	0.575
1719000	0.717	0.705	0.717	0.727	0.741
1769000	0.842	0.832	0.842	0.846	0.855
1819000	0.917	0.897	0.917	0.919	0.923
1869000	0.958	0.942	0.958	0.959	0.961
1919000	0.980	0.967	0.980	0.981	0.981
1969000	0.991	0.982	0.991	0.991	0.991
2019000	0.996	0.990	0.996	0.996	0.996
2069000	0.998	0.994	0.998	0.998	0.998
2169000	1.000	0.998	1.000	1.000	1.000
2269000	1.000	0.999	1.000	1.000	1.000
2369000	1.000	1.000	1.000	1.000	1.000

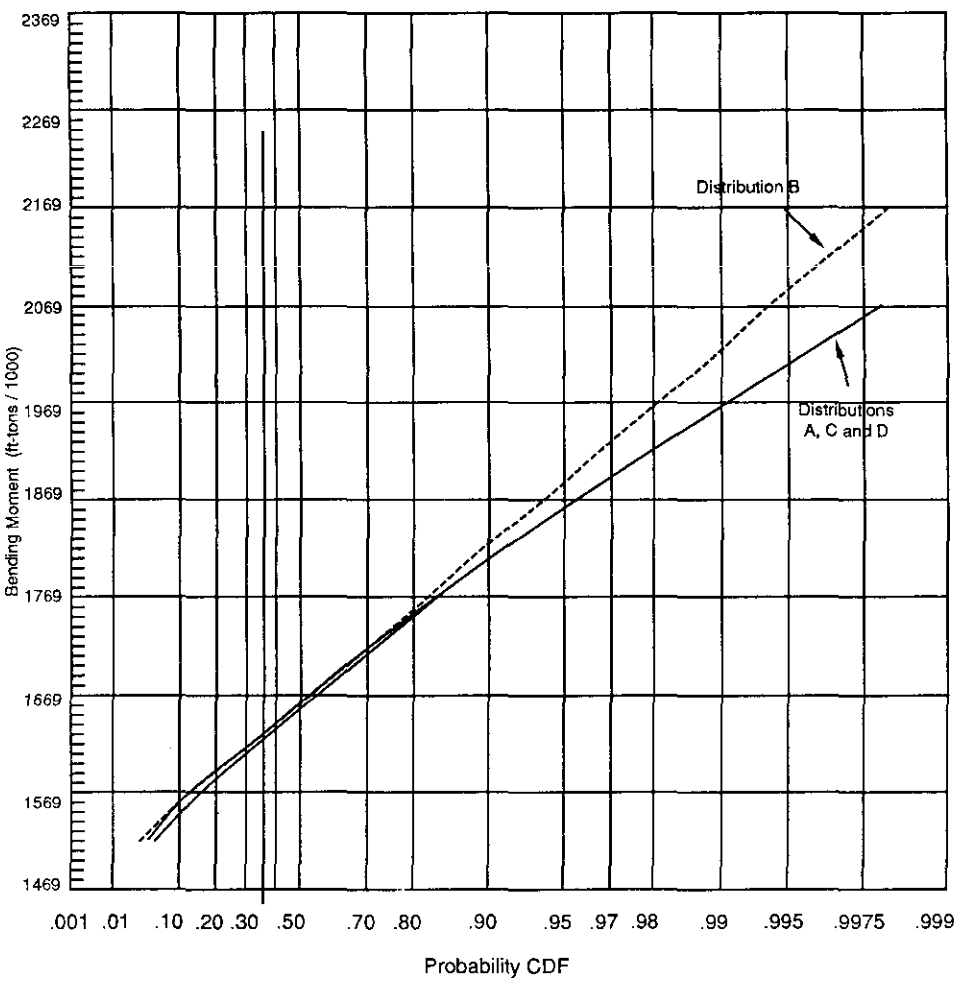

FIGURE 1 - Standard Extremal Variate - Bending Moment on a Tanker

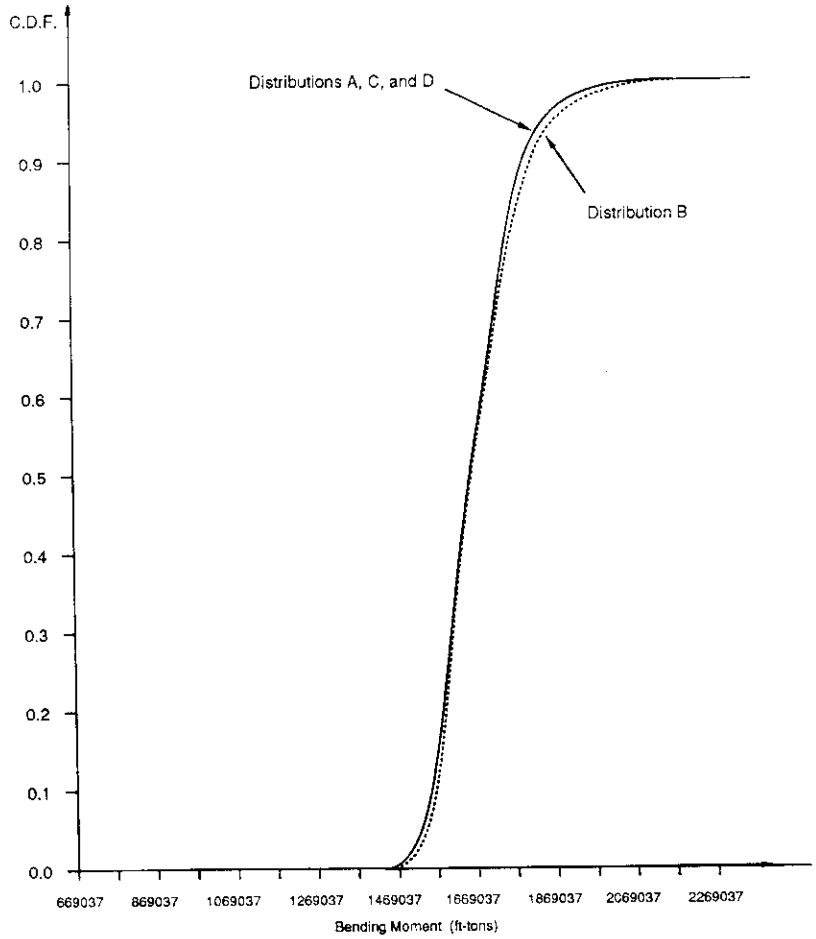

FIGURE 2 - Cumulative Distribution Functions of Extreme Bending Moment on a Tanker

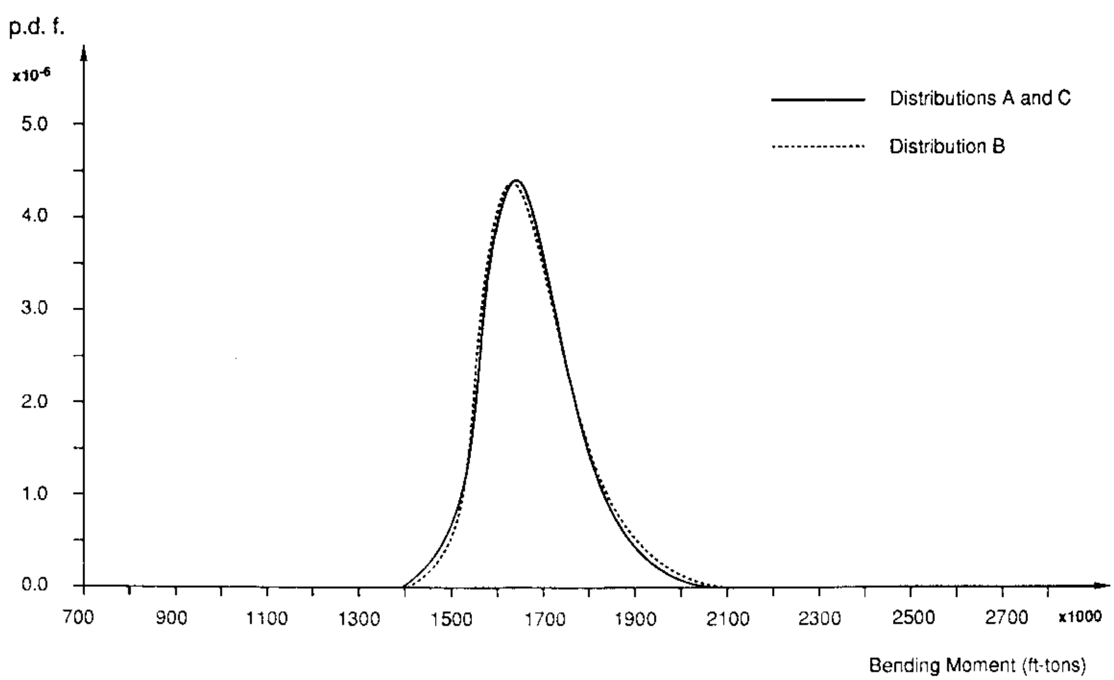

FIGURE 3 - Probability Density Function of Extreme Total Bending Moment on a Tanker