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Reliability Analysis of Offshore Structures

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ABSTRACT

The purpose of this study is to develop a general method for estimating the system reliability of offshore structures with the aid of the full distribution method and to introduce a probabilistic definition of structural redundancy. Structures are treated as systems of structural components. Failure of any number of these components results in a redistribution of the internal or/and external forces. The probability of structural failure is then evaluated by examining a limited number of significant sequences of member failures that produce collapse of the structure. The structure examined is an indeterminate deep offshore truss under fully developed sea conditions. Two different types of material behavior are considered to characterize the type of failure of the components; ductile and brittle behaviors. These reliability analysis and redundancy definition will form an important analytical basis for further investigation of offshore structural integrity.

INTRODUCTION

Recent advances in the probabilistic safety analysis methodology and probability-based design procedures for structures and structural systems have resulted in the publication of a large number of technical reports and papers. Some of these are intended for use in developing probability-based design codes and some in providing a theoretical basis for a risk assessment procedures guide. Typical of the former is an NBS publication by B. Ellingwood et al. (1980) while belonging prominently to the latter is an NRC document by J.W. Hickman et al. (1983).

Although these documents deal primarily with building structures and nuclear power plants, respectively, they address themselves to some of the basic issues of structural integrity assessment and, as such, represent the state-of-the-art in the probabilistic design and analysis of complex engineering systems.

The offshore and ship-building industry has also made significant progress in the same area of engineering endeavor. In fact, a recent symposium/workshop entitled "Design, Inspection and Redundancy" (Faulkner et al., 1984) organized by the Marine Structures Board, National Research Council and held in November 1983, discussed the subject matters indicated in the title within the general framework of reliability analysis and design, and hence is an indication of this industry's recognition that structural reliability issues are a crucial ingredient in design procedures.

While these advances and efforts are impressive, there are still a number of important questions that need to be answered effectively before a probabilistic methodology can truly respond to the needs of the industry. Typically, the following items, which are all heavily interrelated, appear to be in need of immediate attention on the part of practitioners as well as researchers.

A. Estimation of system reliability; reliability estimation procedures must be developed for offshore or ship structures as structural systems.

B. Full distribution methods for improved reliability analysis of offshore or ship structures.

C. Load combination analyses; which load combinations are to be considered in the design, what is the appropriate level of a target safety index for each combination, etc.

D. Effects of structural redundancy on reliability performance; damage-tolerant or fail-safe design concepts must be implemented.

The present study, however, primarily develops a method for estimating the system reliability of offshore structures with the aid of the full distribution method. Also, a probabilistic definition of structural redundancy is introduced in this study. These reliability analysis and redundancy definition will form an important analytical basis for the investigation of the questions surrounding load combination analyses.

EXPECTED MAXIMUM WATER PARTICLE VELOCITY AND ACCELERATION

The wave analysis performed in the present study uses the assumption of the small amplitude (Airy) theory implying that the fluid is inviscid, incompressible and the ratio of wave amplitude to wave length is small. Then, it can be shown that the power spectral density function $S_{\dot{v}\dot{v}}$ of the horizontal component $\dot{v}$ of the water particle velocity is related to the power spectral density function $S_{\eta\eta}(\omega)$ of the water surface elevation η(t) through

$$S_{\dot{v}\dot{v}}(\omega) = \omega^2 \cdot S_{\eta\eta}(\omega)\left[\frac{\cosh[k(z+d)]\cosh[k(z+d)]}{\sinh^2 kd}\right] \quad (1)$$

where ω = circular frequency, k = wave number, d = water depth, z = vertical coordinate axis, positive in the upward direction and measured from the mean sea surface elevation and g = gravity acceleration. Note that

$$\omega^2 = kg \tanh[kd] \quad (2)$$

For deep water, i.e., d $\rightarrow \infty$, Eqs. 1 and 2 respectively reduce to

$$S_{\dot{v}\dot{v}}(\omega) = \omega^2 S_{\eta\eta}(\omega) e^{\frac{2\omega^2}{g}z} \quad (3)$$

and

$$\omega^2 = kg \quad (4)$$

Similarly, the horizontal component $\ddot{v}$ of the water particle acceleration has the spectral density function

$$S_{\ddot{v}\ddot{v}}(\omega) = \omega^2 S_{\dot{v}\dot{v}}(\omega) = \omega^4 S_{\eta\eta}(\omega) e^{\frac{2\omega^2}{g}z} \quad (5)$$

For the purpose of this study, we assume that the offshore tower is excited by waves under fully developed sea conditions for which the Pierson-Moskowitz spectrum of the following form is used (Pierson & Moskowitz, 1964);

$$S_{\eta\eta}(\omega) = \frac{\alpha g^2}{\omega^5} \cdot exp[-\beta(\frac{g}{\omega W})^4] \qquad 0 \le \omega < \infty \quad (6)$$

where the parameters α and β are assumed to be α = 0.0081 and β = 0.74 in the numerical analysis that follows. The quantity W in Eq. 6 indicates a windspeed representative of a fully developed sea condition. The spectral density function for W = 1160 in/sec (29.46 m/sec) is depicted in Fig. 1. In the present analysis, W is treated as a random variable governed by a log-normal distribution function.

It follows from random process theory (Cramér & Leadbetter, 1958) that the expected maximum value of $|\dot{v}(t)|$ in $0 \le t \le T$ is given in approximation by

$$\dot{v}_{max} \doteq E[max|\dot{v}(t)| \quad in \quad 0 \le t \le T] = K_{\dot{v}}\sigma_{\dot{v}} \quad (7)$$

where

$$K_{\dot{v}} = \sqrt{2\ln[2\dot{\nu}^+(0)T]} + \frac{\gamma}{\sqrt{2\ln[2\dot{\nu}^+(0)T]}} \quad (8)$$

and

$$\gamma = 0.5772 \quad \text{(Euler's constant)} \tag{9}$$

In Eq. 8, T is the duration of the storm, $\dot{\nu}^+(0)$ is the expected rate of zero crossing from below by the velocity process $\dot{v}(t)$ and is given by

$$\dot{\nu}^+(0) = \frac{1}{2\pi} \cdot \frac{\sigma_{\ddot{v}}}{\sigma_{\dot{v}}} \tag{10}$$

where $\sigma_{\dot{v}}$ and $\sigma_{\ddot{v}}$ are the standard deviation of $\dot{v}(t)$ and $\ddot{v}(t)$ and are obtained from integrating their respective power spectral density functions;

$$\sigma_{\dot{v}}^2 = \int_0^\infty S_{\dot{v}\dot{v}}(\omega)\, d\omega \tag{11}$$

and

$$\sigma_{\ddot{v}}^2 = \int_0^\infty S_{\ddot{v}\ddot{v}}(\omega)\, d\omega \tag{12}$$

Similarly,

$$\ddot{v}_{max} \doteq E[max|\ddot{v}(t)| \quad in \quad 0 \le t \le T] = K_{\ddot{v}}\sigma_{\ddot{v}} \tag{13}$$

with

$$K_{\ddot{v}} = \sqrt{2\ln[2\ddot{\nu}^+(0)T]} + \frac{\gamma}{\sqrt{2\ln[2\ddot{\nu}^+(0)T]}} \tag{14}$$

and

$$\ddot{\nu}^+(0) = \frac{1}{2\pi} \cdot \frac{\sigma_v}{\sigma_{\ddot{v}}} \tag{15}$$

where $\ddot{\nu}^+(0)$ is the expected rate of zero crossing from below by the acceleration process $\ddot{v}(t)$ and

$$\sigma_v^2 = \int_0^\infty S_{vv}(\omega)\, d\omega \tag{16}$$

with

$$S_{vv}(\omega) = \omega^2 S_{\ddot{v}\ddot{v}}(\omega) = \omega^6 S_{\eta\eta}(\omega) e^{\frac{2\omega^2}{g}z} \tag{17}$$

In order to avoid undue analytical complications, the dependence of the power spectral density functions $S_{\dot{v}\dot{v}}(\omega)$, $S_{\ddot{v}\ddot{v}}(\omega)$ and $S_{vv}(\omega)$ on z is simplified. This is accomplished by using

$$\omega_m = (\frac{4}{5}\beta)^{1/4} \cdot \frac{g}{W} \tag{18}$$

in the factor $\exp(2\omega^2 z/g)$ in Eqs. 3, 5 and 16. Hence, the values of these power spectral density functions decrease in the form of a negative exponential function $\phi(z)$ as the water depth -z increases.

$$\phi(z) = exp[\frac{2\omega_m^2 z}{g}] \tag{19}$$

Furthermore, in evaluating $\sigma_{\dot{v}}$, $\sigma_{\ddot{v}}$ and σ_v, the respective integrations (Eqs. 11, 12 and 16) will be carried out numerically up to $\omega = \omega_u = 24$ rad/sec. That ω_u covers the frequency range over which the power spectral density $S_{\eta\eta}(\omega)$ is significant can be seen from the fact that $\omega_m = 0.289$ rad/sec for a mean windspeed of W = 1160 in/sec (29.46 m/sec). Note here, that the value of ω_u was taken to be extremely large but the same numerical results would be obtained using a smaller value (i.e., 6 rad/sec)

WIND-INDUCED WAVE FORCES

While the structure we deal with in this study is a fixed offshore truss as shown in Fig. 3, we will first consider vertically standing offshore piles in order to evaluate the effect of wind-induced wave forces on truss structures. The wave force on an offshore pile is usually estimated from the well-known empirical formula suggested by Morrison et al. (1950):

$$f(z) = C_m\rho\frac{\pi D^2}{4}\ddot{v}(z) + \frac{1}{2}C_D\rho D\dot{v}(z)|\dot{v}(z)| \tag{20}$$

in which t = time, f(z) = horizontal force/unit length of the pile at water depth z, ρ = mass density of the water, D = pile diameter, and C_m and C_D are respectively the inertial and drag coefficients. In the numerical analysis that follows, $C_m = 1.5$ and $C_D = 1.0$ are assumed.

In the dynamic analysis of pile response, the interaction between wave and structure should be considered when the velocity and acceleration of the structural motion are of the same order of magnitude as that of the water particles. It is generally accepted that the effect of this interaction can be incorporated into the Morrison equation by using the instantaneous relative velocity and acceleration between the structure and water particles. In the present study, however, this effect is disregarded and the structural analysis is performed in a quasi-static fashion. In a recent work (Paliou, C. et al., 1986) this approach is extended to dynamic response evaluation, without disregarding this effect, and including a fatigue analysis and the effect of inspection.

In order to circumvent undue analytical difficulty and at the same time to be on the conservative side, $\dot{v}_{max}\,exp(\frac{\omega_m^2}{g}z)$ and $\ddot{v}_{max}\,exp(\frac{\omega_m^2}{g}z)$, evaluated in the preceding section, are used in Eq. 20. This is obviously a conservative approximation since in actuality the maximum values of $\dot{v}(t)$ and $\ddot{v}(t)$ will not usually occur at the same time instant, but also bearing in mind that the main subject of this paper is the development of a reliability analysis procedure, this approximation of the load configuration serves as an illustration. Hence,

$$f(z) = C_m\rho\frac{\pi D^2}{4}\ddot{v}_{max}\,exp(\frac{\omega_m^2}{g}z) + \\ + \frac{1}{2}C_D\rho D\dot{v}_{max}^2\,exp(\frac{2\omega_m^2}{g}z) \qquad (21)$$

The force per unit length evaluated by Eq. 21 is based on the Morrison formula for vertically standing members. Even when a member is inclined with respect to the vertical direction by a small angle θ, we assume that Eq. 21 can still be used with $\dot{v}_{max}\cos\theta$ and $\ddot{v}_{max}\cos\theta$ in place of $\dot{v}_{max}$ and $\ddot{v}_{max}$, respectively. This assumption basically indicates that $\dot{v}_{max}\cos\theta$ and $\ddot{v}_{max}\cos\theta$ are assumed to produce (in approximation) forces per unit length perpendicular to the member axis. The effect of the components of $\dot{v}_{max}$ and $\ddot{v}_{max}$ in the direction parallel to the member axis is disregarded in approximation. The horizontal component of the resultant force derived from the distributed force acting along the member and in the perpendicular direction thereof is divided into two equal components, and each is considered as an external force acting on each end (a node of the truss structure to be analyzed) of the member. The structure is then subjected to those external forces resulting from the distributed forces acting on all of its members; F_i represents the sum of these forces acting at node i (Fig. 3).

As mentioned in the previous section, the windspeed W that represents a fully developed sea state is assumed to be a random variable governed by a log-normal distribution (Yang, 1978)

$$f_W(w) = \frac{2\log e}{\sqrt{2\pi}\sigma_W^* w}\cdot exp\{-\frac{1}{2}[\frac{\log(C_1 w^2) - \mu_W^*}{\sigma_W^*}]^2\} \qquad (22)$$

in which w is measured in in/sec, μ_W^* and σ_W^* represent the expected value and standard deviation of $\log Y_{1m} = \log C_1 W^2$, where Y_{1m} is the annual expected maximum wave height and

$$C_1 = \frac{3.85}{2g}\sqrt{\frac{\alpha}{\beta}} \qquad (23)$$

For the North Sea, the use of $\mu_W^* = 2.842$ and $\sigma_W^* = 0.1$ was suggested in Yang and Freudenthal (1977). The density function of W with these parameter values is depicted in Fig. 2.

The expected value of μ_W and standard deviation σ_W of W can then be evaluated from

$$\mu_W = exp(\mu + \frac{\sigma^2}{2}) \qquad (24)$$

and

$$\sigma_W = \mu_W\sqrt{e^{\sigma^2} - 1} \qquad (25)$$

where

$$\mu = (\mu_W^* - \log C_1)\cdot\frac{\ln 10}{2} \qquad (26)$$

$$\sigma = \sigma_W^*\cdot\frac{\ln 10}{2} \qquad (27)$$

The reliability analysis then proceeds as follows:

(a) Working in the range of windspeed from a mean value of W - 6 standard deviations ($\mu_W - 6\sigma_W$) to a mean value of W + 6 standard deviations ($\mu_W + 6\sigma_W$), the range is divided into 200 intervals and the probability of windspeed in each interval is evaluated.

(b) Using the Pierson-Moskowitz spectrum (Eq. 6), the expected maximum wave particle velocity and acceleration ($\dot{v}_{max}$ and $\ddot{v}_{max}$) which corresponds to each windspeed is computed.

(c) Having computed the values of the expected maximum water particle velocity and acceleration, the distribution load exerted by the waves on each member is computed by the Morrison equation (Eq. 21). Thus, the horizontal components F_i of the wave force acting on each node of the structure can be calculated.

(d) Two cases are examined herein. Case I - *Ductile Behavior*: Whenever a number of members fails either in tension, compression or buckling, the effect of those members will be replaced by pairs of external forces acting in the direction of the axes of these members equal to the member forces due to the same loading condition of the intact structure (see Fig. 4a). Case II - *Brittle Behavior*: Whenever a number of members fails either in tension, compression or buckling, no external force will be assumed to act in the direction of the axes of these members as is considered in Case I (see Fig. 4b). The external forces F_i (due to waves) remain to act at all the nodes. The stresses in the members which are still intact must therefore be re-evaluated under these loading conditions. In this process of re-evaluation, those members which have failed do not contribute to the construction of the stiffness matrix.

(e) For a number of windspeeds w, the conditional probability of failure of the structure is computed. This computation, however, is quite involved since, in principle, we must consider all the possible sequences of member failures that lead to collapse of the structure. In the present study, a structural collapse is considered to have occurred when excessive structural deflections materialize after failure of a number of members or the corresponding stiffness matrix becomes at least near singular.

(f) At this point, the notion of simultaneous failure of members used in this study shall be made clear. It is acknowledged that if the intensity of the load increases from zero to a certain level, then the probability of simultaneous failure of two or more members will be zero if the strengths of the members are random. However, for ease of the probabilistic analysis, it is assumed throughout this study that simultaneous failures can take place in those members whose strengths are less than the internal forces resulting from this level of load intensity.

(g) In the evaluation of this conditional probability mentioned in (e) above, the material strength such as σ_{Yt}, σ_{Yc} or σ_{Bk} is assumed to be a random variable governed by a normal distribution with mean values 36 ksi (248.04 MPa) for σ_{Yt}, σ_{Yc} and $\pi^2 EI_k/(L_k^2 A_k)$ for σ_{Bk}, with various values of the coefficient of variation.

(h) In order to evaluate the conditional probability of failure $P_f(w)$, the procedure described in the next section is used. At this point, it should be noted that the branch and bounding operations appearing originally in the work of Murotsu and Okada (1981) contained certain approximations that may not be valid in certain circumstances. It is the purpose of this paper to offer a more detailed and accurate presentation of this method, thus improving the original development.

(i) Finally, the unconditional probability of failure P_F of the structure is evaluated as

$$P_F = \int_0^\infty P_f(w) f_W(w)\, dw \tag{28}$$

RELIABILITY ANALYSIS OF REDUNDANT STRUCTURES

The probability $G_i^{(0)}$ of the event that member i will fail first, while no other members have failed is

$$G_i^{(0)} = F_i^{(0)} \prod_{\substack{j=1 \\ j\neq i}}^{n} F_j^{*(0)} \tag{29}$$

where

$F_i^{(0)}$ = probability of failure of member i (30)

$F_j^{*(0)}$ = probability of survival of member j (31)

n = number of members in the structure (32)

The probability $SG_k^{(i)}$ of the event that member k fails after member i has failed first, while all the remaining members survive is

$$SG_k^{(i)} = G_i^{(0)} \cdot G_k^{(i)} \tag{33}$$

where

$$G_k^{(i)} = F_k^{(i)} \prod_{\substack{j=1 \\ j\neq i,k}}^{n} F_j^{*(i)} \tag{34}$$

with

$F_k^{(i)}$ = probability that member k will fail under redistribution of the load immediately after the event that member i and only member i has failed.

and

$F_k^{*(i)}$ = probability that member k will survive under redistribution of the load immediately after the event that member i and only member i has failed.

Furthermore

$$SG_l^{(i,k)} = G_i^{(0)} G_k^{(i)} G_l^{(i,k)} \tag{35}$$

where

$SG_l^{(i,k)}$ = probability that member l fails after member i fails first and member k fails second.

$G_l^{(i,k)}$ = given the failure of members i and k, the probability that member l fails while all other members (except members i, k, l) survive.

Similar definitions apply to $SG_m^{(i,k,l)}$, $SG_n^{(i,k,l,m)}$, etc.

At this point, it should be noted that the probability $F_k^{(i)}$ is conditional due to the fact that member k has already survived $\sigma_k^{(0)}$ (i.e., stress of member k in the intact structure). In order to compute, in general, the probability $F_k^{(i_1,i_2,\ldots,i_n)}$ (i.e., assuming a sequential failure of members $i_1 \to i_2 \to \ldots \to i_n$) the following expression is used

$$F_k^{(i_1,\ldots,i_n)} = \begin{cases} \dfrac{\hat{F}_k^{(i_1,\ldots,i_n)} - F_k^{(i_1,\ldots,i_m)}}{F_k^{*(i_1,\ldots,i_m)}} & \text{for condition (a)} \\ 0 & \text{for condition (b)} \\ \hat{F}_k^{(i_1,\ldots,i_n)} & \text{for condition (c)} \end{cases} \tag{36}$$

where

Condition (a) : $\sigma_k^{(i_1,\ldots,i_m)} = max\{|\sigma_k^{(i_1,\ldots,i_l)}| : \sigma_k^{(i_1,\ldots,i_l)} \cdot \sigma_k^{(i_1,\ldots,i_n)} > 0\}$

Condition (b) : $\{|\sigma_k^{(i_1,\ldots,i_n)}| < |\sigma_k^{(i_1,\ldots,i_l)}| : \sigma_k^{(i_1,\ldots,i_l)} \cdot \sigma_k^{(i_1,\ldots,i_n)} > 0\}$

Condition (c) : $\{\sigma_k^{(i_1,\ldots,i_n)} \cdot \sigma_k^{(i_1,\ldots,i_l)} < 0\}$

for all l = 1,. . ., n-1

and

$$\hat{F}_k^{(i_1,\ldots,i_n)} = Prob\{\sigma_k^{(i_1,\ldots,i_n)} > \sigma_Y\} \tag{37}$$

Consider now a set of k_p members $(r_1, r_2, \ldots, r_{k_p})$ which are identified as combination p and whose failures in any sequence (out of a possible $k_p!$ sequences) will produce system failure. Assume further that there are a total of m such combinations of members. Let r_{pqj} $(j = 1, 2, \ldots, k_p)$ identify the member which fails j-th in sequence q when combination p is considered. For example, suppose there is a truss consisting of n=4 members; all these members are identified by a member ID number 1, 2, 3 and 4. Suppose further, that the failure of two members will produce collapse in this truss. This means that there are ${}_4C_2 = 6$ combinations of members whose failure will result in collapse. The first combination consisting of the two members 1 and 2 is identified by combination ID number p=1, and since this combination involves two members, $k_1 = 2$. The second combination is identified by p=2, consisting of the two members 1 and 3 and for this combination, $k_2 = 2$, and so on. Also, for each combination, there are $k_p! = 2! = 2$ sequences according to which the two members of that combination can fail. For instance, if combination ID number p=1, q=1 identifies the sequence of failure $1 \rightarrow 2$, while if q=2, the sequence $2 \rightarrow 1$. Therefore, $r_{121} = 2$ and $r_{122} = 1$ in this example.

Writing $EF_i^{(j)}$ for the event that member i fails following the failure of (j-1) other members,

$$EFS_{pq} = EF^{(1)}_{r_{pq1}} \;\; EF^{(2)}_{r_{pq2}} \;\; \ldots \;\; EF^{(k_p)}_{r_{pqk_p}} \qquad (38)$$

Using the notation introduced above, we note that

$$Prob\{EFS_{pq}\} = SG^{(r_{pq1}, r_{pq2}, \ldots, r_{pq(k_p-1)})}_{r_{pqk_p}} \qquad (39)$$

The failure probability of a redundant structure can now be written as

$$P_f = Prob\{\bigcup_{p=1}^{m} \bigcup_{q=1}^{k_p!} EFS_{pq}\} \qquad (40)$$

The upper bound of the failure probability can be computed either by

$$P_{f_U} = \sum_{i=1}^{n} Prob\{EF_i^{(1)}\} \qquad (41)$$

or by

$$P_{f_U} = \sum_{p=1}^{m} \sum_{q=1}^{k_p!} Prob\{EFS_{pq}\} \qquad (42)$$

These two formulations are not used in the present study, however, because the first is too conservative without discriminating between redundant and non- redundant structures and the second requires enumeration of all possible failure modes, and thus is usually too time-consuming. An alternative upper bound is evaluated by

$$P_{f_U} = \sum\nolimits^{*} Prob\{EFS_{pq}\} \qquad (43)$$

where $\sum^*$ signifies summation over the selected dominant modes of failure. We assume that there are m' of them.

The lower bound of failure probability can be computed as

$$P_{f_L} = max_{[p,q]} Prob\{EFS_{pq}\} \qquad (44)$$

where the maximum is taken over all the pairs of p and q that are examined, and each pair represents a particular failure mechanism and a particular sequential path of member failures.

Following Murotsu and Okada (1981), three steps, which consist of branching operations, upper and lower bound adjustments and bounding operations are considered, in order to evaluate P_{f_U} and P_{f_L}.

Step 1 : Branching Operations - Selection of Dominant Modes

A combination of members and their particular failure sequence, or a pair of p and q, are selected so that it yields a failure mechanism with the largest (structural) failure probability among possible pairs of p and q. To this end, first identify member r_{pq1} such that

$$Prob\{EF^{(1)}_{r_{pq1}}\} = G^{(0)}_{pq1} = max_{[i_1 \in I_{s_1}]} Prob\{EF^{(1)}_{i_1}\} \qquad (45)$$

where $Prob\{EF^{(1)}_{i_1}\} = G^{(0)}_{i_1}$ is the probability that member i_1 and only member i_1 will fail under the prescribed loading condition. Hence, the probability that all other members will remain intact must be taken into consideration in evaluating this probability. Set I_{s_1} consists of those members for which $Prob\{EF^{(1)}_{i_1}\} = G^{(0)}_{i_1}$ are larger than a certain prescribed value.

Then, identify r_{pq2} such that

$$Prob\{EF^{(2)}_{r_{pq2}}\} = G^{(r_{pq1})}_{r_{pq2}} = max_{[i_2 \in I_{s_2}]} Prob\{EF^{(2)}_{i_2}\} \tag{46}$$

where $Prob\{EF^{(2)}_{i_2}\} = G^{(r_{pq1})}_{i_2}$ is the probability of failure of member i_2 and only member i_2 after the redistribution of internal forces immediately subsequent to the failure of member r_{pq1}. Set I_{s_2} consists of those members i_2 for which $Prob\{EF^{(2)}_{i_2}\}$ are greater than a prescribed value.

Proceeding similarly and examining if structural failure occurs at the end of each member failure, the particular combination p' and particular sequence q' which produce structural failure will be identified together with sets $I_{s_1}, I_{s_2}, \ldots, I_{s_{k_{p'}}}$. Then, the corresponding structural failure probability is

$$Prob\{EFS_{p'q'}\} = Prob \prod_{j=1}^{k_{p'}} \{EF^{(j)}_{r_{p'q'j}}\} \tag{47}$$

Step 2 : Adjustments of Upper and Lower Bounds

Define P_{f_L} and $P_{f_U}(1)$ as

$$P_{f_L} = P_{f_L}(1) = Prob\{EFS_{p'q'}\} \tag{48}$$

and

$$P_{f_U}(1) = P_{f_L} \tag{49}$$

Then, find the second combination p'' and q'' with the system failure probability $Prob\{EFS_{p''q''}\}$ and update $P_{f_U}(1)$ and $P_{f_L}(1)$ so that

$$P_{f_U}(2) = P_{f_U}(1) + P_{f_L}(2) \tag{50}$$

where $P_{f_L}(2) = Prob\{EFS_{p''q''}\}$ and

$$P_{f_L} = P_{f_L}(2) \tag{51}$$

if $P_{f_L}(2) \geq P_{f_L}(1)$. Repeat the procedure m' times until all the dominant modes have been examined. Then $P_{f_U}(m')$ becomes the upper bound of the failure probability of the system.

Step 3 : Bounding Operations

To find p'' and q'', the following procedure is followed. First, consider members $i_{k_{p'}}$, in the set $I_{s_{k_{p'}}}$ except $r_{p'q'k_{p'}}$, and examine if $k_{p''}$ members consisting of $r_{p'q'1}$ $(= r_{p''q''1})$, $r_{p'q'2}$ $(= r_{p''q''2})$, $\ldots$,$r_{p'q'(k_{p'}-1)}$ $(= r_{p''q''(k_{p'}-1)})$, $r_{p''q''k_{p'}}$, $\ldots$,$r_{p''q''k_{p''}}$ represent a failure combination p'' in a particular sequence q'', where $k_{p''} \geq k_{p'}$. The structural failure probability associated with this particular pair of p'' and q'' will be evaluated only when

$$Prob\{EF_{r_{p''q''i_{k_{p'}}}}\} > 10^{-a} P_{f_L} \tag{52}$$

where $i_{k_{p'}} \in I_{s_{k_{p'}}}$ but $i_{k_{p'}} \neq r_{p'q'k_{p'}}$. If Eq. 52 is satisfied, then the combination p'' and q'' represents a dominant failure mode.

The above procedure is repeated until all the possible pairs of p and q are exhausted. Note, however, that Eq. 52 limits to a minimum the number of failure modes for which the structural failure probabilities are to be computed.

Computer codes have been developed to implement the analytical procedures indicated above and numerical examples have been worked out using the structure shown in Fig. 3

The upper and lower bounds evaluated with the aid of the procedure described above are still conditional to a specific windspeed and corresponding loading condition, and hence they should actually be denoted by P_{f_U}(w) and P_{f_L}(w), respectively.

In the present study, we use P_{f_U}(w) for P_f(w) in approximation, since less complicated numerical examples carried out indicated that these bounds are quite close particularly in the range of high windspeeds where the conditional probability values become more crucial.

DEFINITION OF STRUCTURAL REDUNDANCY

There are a number of definitions for structural redundancy ranging from that implied by the well-known degree of structural indeterminacy in structural analysis to those listed below, as suggested by Lloyd and Clawson (1984)

Redundant Factor =

$$= \frac{\text{intact strength}}{\text{intact strength-damaged strength}}$$

Reserve Resistance Factor =

$$= \frac{\text{environmental load at collapse (undamaged)}}{\text{design environmental load}}$$

Residual Resistant Factor =

$$= \frac{\text{environmental load at collapse (damaged)}}{\text{environmental load at collapse (undamaged)}}$$

In the present study, however, an attempt is made to define the redundancy by a probabilistic measure. The definition introduced here uses, as a measure of redundancy, the probability P_r^* that the structure will eventually survive, given the failure of one or more (but simultaneously) of its members.

Using the notation introduced in the previous section, the following example attempts to give a frequency interpretation of the redundancy probability defined above.

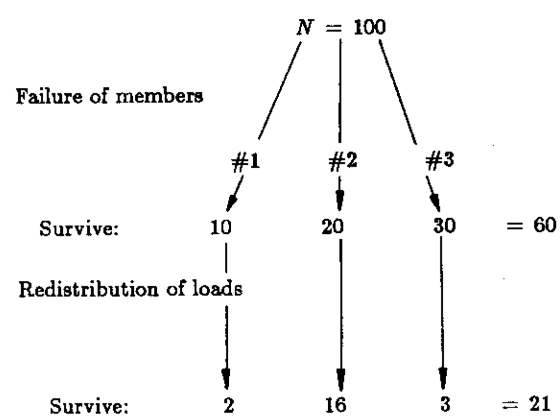
It is supposed that there are initially N nominally identical but statistically different k-member structures. For illustrative purposes, N is assumed to be one hundred (N = 100) and k equal to three (k = 3). Furthermore, it is assumed that the probabilities of failure $G_i^{(0)}$ of members i (i = 1,2,3) of the intact structure due to the initial loading configuration are

$$G_1^{(0)} = 0.1, \qquad G_2^{(0)} = 0.2 \quad \textit{and} \quad G_3^{(0)} = 0.3 \qquad (53)$$

The frequency interpretation of Eq. 53 states that ten (10) out of one hundred (100) structures will suffer from the failure of member 1, twenty (20) out of one hundred structures will suffer from the failure of member 2, thirty (30) out of one hundred structures will suffer from the failure of member 3, but all will survive. After the redistribution of loads in these structures which suffered from the failure of one of their members and survived, it is assumed for illustrative purposes that the probability of survival of those structures has been found to be 0.2, 0.8 and 0.1 for structures without members 1 or 2 or 3, respectively, i.e.

$$\begin{aligned} 1 - G_2^{(1)} - G_3^{(1)} &= 0.2 \\ 1 - G_1^{(2)} - G_3^{(2)} &= 0.8 \\ 1 - G_1^{(3)} - G_2^{(3)} &= 0.1 \end{aligned} \qquad (54)$$

The frequency interpretation of Eqs. 54 now states that two (2) out of the ten (10) structures which survived after the first failure of member 1 will again survive the redistribution of the loads, etc.



makes clear that twenty-one (21) out of the sixty (60) structures which survived the first failure of one of their members, will eventually (i.e., after redistribution of loads) survive. Thus, according to the definition introduced previously, the redundancy probability P_r^* is 21/60 = 0.35.

Explicitely written, Eqs. 53 and 54 lead to

$$P_r^* = \frac{\sum_{i=1}^{k} G_i^{(0)} \cdot \{1 - \sum_{\substack{j=1 \\ j \neq i}}^{k} G_j^{(i)}\}}{\sum_{i=1}^{k} G_i^{(0)}} =$$

$$= \frac{\sum_{i=1}^{k} G_i^{(0)} - \sum_{i=1}^{k} \sum_{\substack{j=1 \\ j \neq i}}^{k} SG_j^{(i)}}{\sum_{i=1}^{k} G_i^{(0)}} =$$

$$= 1 - \frac{\sum_{i=1}^{k} \sum_{\substack{j=1 \\ j \neq i}}^{k} SG_j^{(i)}}{\sum_{i=1}^{k} G_i^{(0)}} \qquad (55)$$

The numerator of the second term on the right-hand side of Eq. 55 is nothing else but the probability of failure of the structure, without including the probability of simultaneous failure of more than one member in the intact structures which, in this example, would produce collapse of the structures.

In order to include this event and to make the definition of the redundancy probability completely general, it should be defined that

$$P_r^* = 1 - \frac{P_f}{\sum_{i=1}^{k} G_i^{(0)} + SF^{(0)}} = 1 - P_f^* \qquad (56)$$

where $SF^{(0)}$ is the summation of the probabilities of the events of simultaneous first failure of more than one member, which lead, however, to survival of the structure. In Eq. 56, P_f^* is defined as the non-redundancy probability, i.e., the probability that the structure will eventually collapse given the first failure of one or more (but simultaneously) of its members.

The probability P_F^* in the numerical example discussed in this study can be written as

$$P_F^* = \int_0^\infty P_f^*(w) f_W(w)\, dw \qquad (57)$$

in which $P_f^*(w)$ is the conditional non-redundancy probability of the structure in a fully developed sea state represented by a windspeed w.

NUMERICAL EXAMPLE

The Pierson-Moskowitz spectrum given in Eq. 6 is used for sea surface elevation η with $\alpha = 0.0081$, $\beta = 0.74$ and g = 386.4 in/sec^2 (9.81 m/sec^2). The spectral density beyond $\omega_u = 24$ rad/sec is disregarded as insignificant. The deep water assumption is used with d = 300 ft (91.5 m) in Eq. 1. This assumption resulted in a simpler expression for the dependence on z of the spectral density functions of the horizontal components of the water particle velocity and acceleration. The duration T of each storm in Eqs. 7 and 13 is assumed to be four (4) hours. While such a duration should also be considered random, the assumption of T being equal to four hours does not appear to be unreasonable.

In employing the Morrison formula in Eq. 21, we assume $C_m = 1.5$, $C_D = 1.0$ and $\rho = 9.61 \times 10^{-8}$ kips·sec^2/in^4 (1.028×10^{-8} N·sec^2/mm^4). The quantity ω_m is the frequency at which the Pierson-Moskowitz spectral density takes a maximum value and is given by Eq. 18; For example, $\omega_m = 0.289$ rad/sec for W = 1160 in/sec (29.46 m/sec). The distribution function of W has a log-normal distribution as shown in Eq. 22 where the constant C_1 is given by Eq. 23 and is equal to 0.0005212 sec^2/ in (0.0205 sec^2/m). We assume that $\mu_W^* = 2.842$ and $\sigma_W^* = 0.1$. These correspond to $\mu_W = E[W] = \bar{W} = 1160$ in/sec (29.46 m/sec) and V_W (coefficient of variation of W) $\doteq$ 0.1.

The truss considered for the numerical example is shown in Fig. 3 and its geometrical and other characteristics are listed in Table 1. The material strengths σ_{Yt}, σ_{Yc} and σ_{Bk} are assumed to be normally distributed random variables as mentioned in the previous section. Also, their values are statistically independent from member to member.

The conditional probability $P_f(w)$ (upper bound) is evaluated for several windspeeds: W = 621 in/sec (15.77 m/sec), 894 in/sec (22.71 m/sec), 1160 in/sec (29.46 m/sec), 1425 in/sec (36.2 m/sec), 1698 in/sec (43.13 m/sec), 1963 in/sec (49.86 m/sec), thus covering the range from μ_W - 4 σ_W to μ_W + 6 σ_W. The values of $P_f(w)$ at these wind velocities w for Cases I and II are plotted in Fig. 5a.

The unconditional probability of failure P_F is then obtained with the aid of Eq. 28. The integrand $P_f(w)f_W(w)$ for both Cases I and II appears in Fig. 5b. Constructing such a plot, the values of $P_f(w)$ other than those already computed are obtained by interpolation. As can be seen from the plots of $P_f(w)f_W(w)$ on log-scale, the contribution towards P_F from lower windspeeds is negligible. The final results indicate that $P_F = 0.24 \times 10^{-3}$ (Case I) and 0.88×10^{-3} (Case II) for this truss.

With respect to the conditional non-redundancy probability $P_f^*(w)$, the denominator of the second term in Eq. 56 is computed at the same windspeed values as those used for the computation of $P_f(w)$. The results for P_f^* (w) are shown for both Cases I and II in Table 2 and are plotted in Fig. 6a. Fig. 6b displays the values of the integrand of Eq. 57. Finally, P_F^* is computed as 0.99 $\times 10^{-2}$ for Case I and 0.93 $\times 10^{-1}$ for Case II and the corresponding unconditional redundancy probability P_R^* is equal to 0.99 for Case I and 0.91 for Case II.

CONCLUSIONS

An analytical method, numerical procedure and computer codes are developed to evaluate the probability of structural failure. While the structure considered is a fixed offshore structure, the methodology is general enough to be extended to other types of offshore structures. Crucially important in the methodology is the fact that it treats the structure as a system of structural components with the understanding that failure of these components in any number results in redistribution of the internal (and possibly external) forces.

The external forces that act on the offshore structure stem from wind-induced waves. These forces are evaluated under the assumption of the small amplitude wave theory together with the assumption that the flow is irrotational and inviscid.

The horizontal components of the water particle velocity and acceleration the offshore structure is subjected to during each storm are derived from the Pierson-Moskowitz spectrum for the sea surface elevation. In its analytical form, the spectrum contains a mean windspeed value representative of each storm, which is assumed to last four hours. Assuming further that the sea surface elevation is a Gaussian random process, the expected maximum values of the horizontal water particle velocity and acceleration in their absolute values are evaluated with the aid of random process theory.

The attenuation of these expected maximum values along the water depth depends on the frequency. For simplicity of analysis, this dependence is disregarded and a form of negative exponential attenuation is used for all the frequency components. Also, disregarding the effect of relative motion between a water particle and the offshore structure, the Morrison equation is used to compute the external forces acting on the truss members, taking into consideration the angle of inclination of these members with respect to the vertical direction and transforming these distributed forces into concentrated forces acting on the nodes of the truss.

The probability of structural failure is evaluated by examining only a limited number of higher-probability sequences of member failures that produce collapse of the structure. The probability analysis involved could become quite time-consuming if all the possible sequences leading to structural collapse were considered and if the structure consists of a large number of structural components.

The probability analysis is important not only because it will provide us with the probability of structural failure but also because it will make it possible to define the redundancy probabilistically, for example, as the probability of structural survival given the first failure of one of the members.

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TABLE 1. Geometrical and Material Characteristics of Structural Components

#	Length (ft)	Diameter (in)	Thickness (in)	Area (in^2)	Inertia Mom. (in^4)
1	105.5	50.1	2.0	302.2	87554
2	80.4	50.1	2.0	302.2	87554
3	70.3	50.1	2.0	302.2	87554
4	45.2	50.1	2.0	302.2	87554
5	105.5	50.1	2.0	302.2	87554
6	80.4	50.1	2.0	302.2	87554
7	70.3	50.1	2.0	302.2	87554
8	45.2	50.1	2.0	302.2	87554
9	120.0	45.0	1.0	138.2	33469
10	151.7	45.0	1.0	138.2	33469
11	151.7	45.0	1.0	138.2	33469
12	121.2	36.0	0.75	83.1	12906
13	121.2	36.0	0.75	83.1	12906
14	103.3	36.0	0.75	83.1	12906
15	103.3	36.0	0.75	83.1	12906
16	78.6	36.0	0.75	83.1	12906
17	78.6	36.0	0.75	83.1	12906
18	60.0	27.0	1.25	101.1	8401

Modulus of Elasticity = 29000 ksi

σ_{Yc}, σ_{Yt} = 36 ksi

Note : 1 ft = 0.3048 m ; 1 in = 25.4 mm ; 1 ksi = 6.89 MPa.

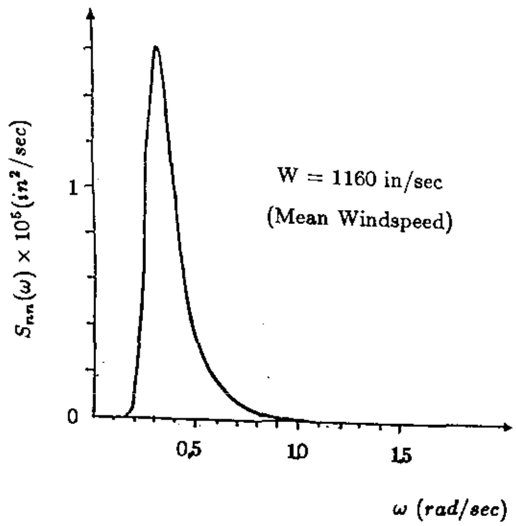

FIG. 1 One-Sided Spectral Density of Ocean Wave Elevation.

TABLE 2. Conditional Probabilities $P_f(w)$ and $P_f^*(w)$ Due to Various Windspeed Values

Windspeed (in/sec)	$G_i^{(0)} + SF^{(0)}$	$P_f(w)$ Case I
621.22	$0.591 \cdot 10^{-5}$	$0.309 \cdot 10^{-10}$
894.51	$0.553 \cdot 10^{-4}$	$0.199 \cdot 10^{-8}$
1159.76	$0.807 \cdot 10^{-3}$	$0.339 \cdot 10^{-6}$
1425.01	$0.125 \cdot 10^{-1}$	$0.270 \cdot 10^{-2}$
1698.30	0.141	$0.391 \cdot 10^{-1}$
1963.56	0.624	0.366

$P_f^*(w)$ Case I	$P_f(w)$ Case II	$P_f^*(w)$ Case II
$0.523 \cdot 10^{-5}$	$0.144 \cdot 10^{-9}$	$0.244 \cdot 10^{-4}$
$0.359 \cdot 10^{-4}$	$0.947 \cdot 10^{-7}$	$0.171 \cdot 10^{-2}$
$0.420 \cdot 10^{-3}$	$0.517 \cdot 10^{-4}$	$0.641 \cdot 10^{-1}$
0.216	$0.419 \cdot 10^{-2}$	0.335
0.277	0.141	0.997
0.587	0.624	1.000

Note : 1 in = 25.4 mm

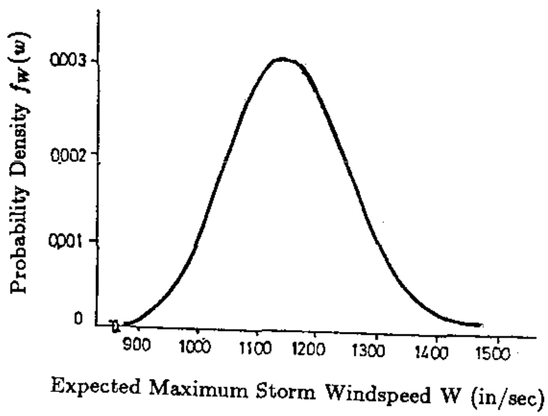

FIG. 2 Probability Density Function of Expected Maximum Storm Windspeed.

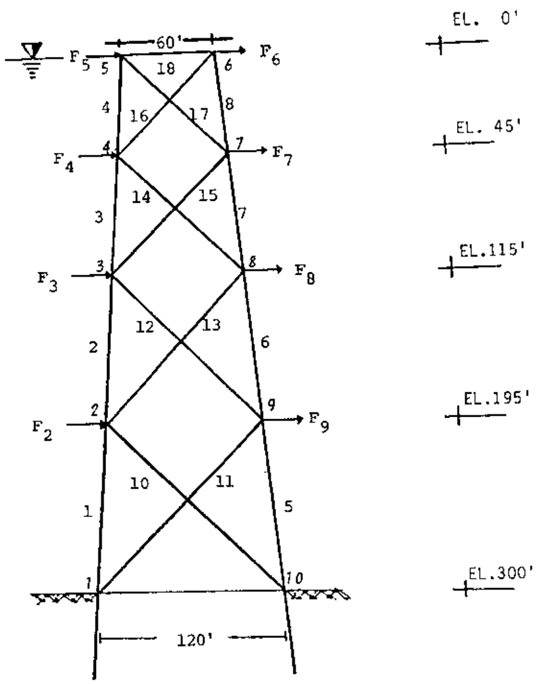

FIG. 3 Intact Structure.

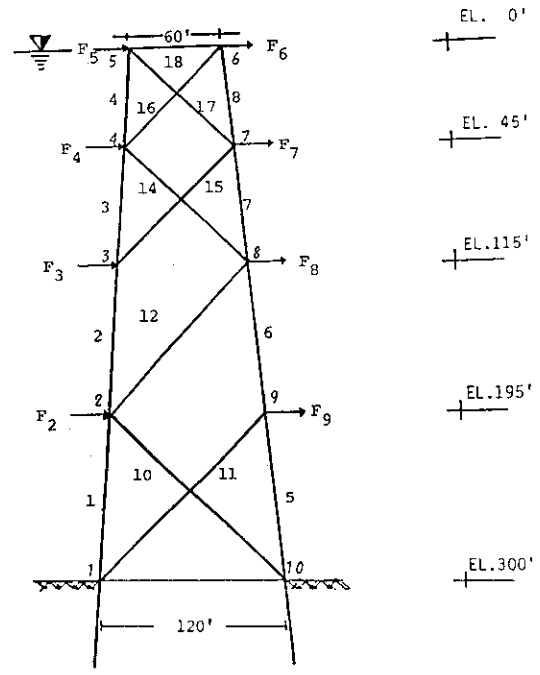

FIG. 4b Loading Condition Subsequent to Failure of Member 13 (Case II - *Brittle Behaviour*).

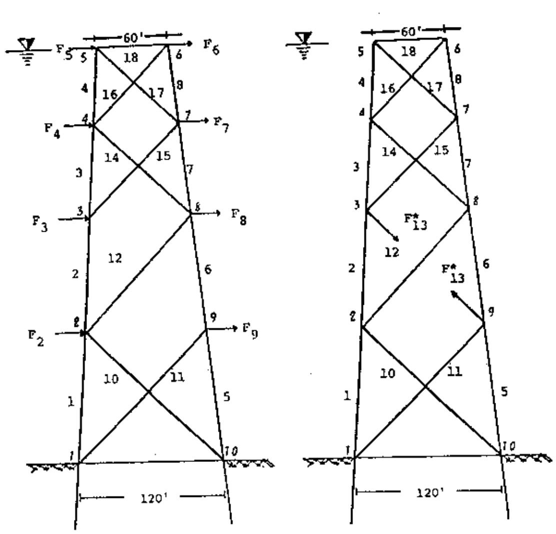

FIG. 4a Loading Condition Subsequent to Failure of Member 13 (Case I - *Ductile Behaviour*).

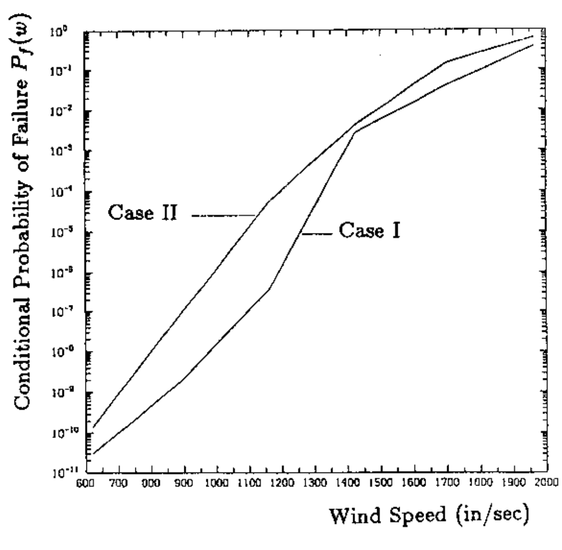

FIG. 5a Conditional Probability of Failure $P_f(w)$ vs. Windspeed.

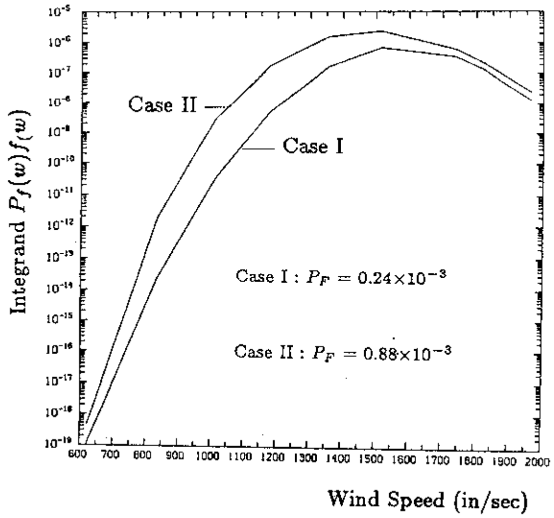

FIG. 5b Integrand $P_f(w)f_W(w)$ vs. Windspeed.

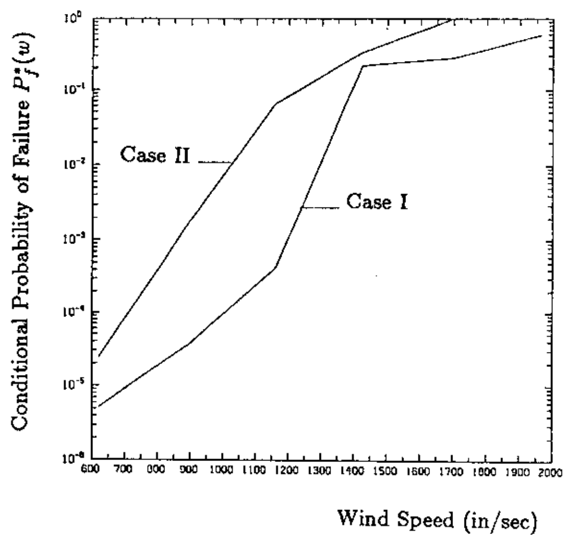

FIG. 6a Conditional Probability of Failure $P_f^*(w)$ vs. Windspeed.

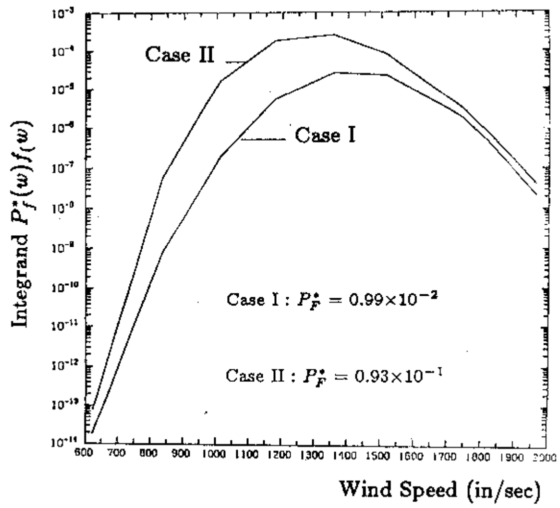

FIG. 6b Integrand $P_f^*(w)f_W(w)$ vs. Windspeed.