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# Methodologies for the Selection of Safety Factors and Safety Levels in the Design of Marine Structures

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ABSTRACT

Structural design optimization is taken to mean the minimization of the expected present value of the sum of the initial cost of the structure and the losses incurred due to the structure entering a certain limit state. Expressions are derived for the optimal central safety factor and for its associated probability of failure as a function of the "importance ratio" of the structure.

Current limit state design codes use a discrete number of safety levels. Herein a methodology is proposed for the selection of these safety levels and to assess their applicability using a one dimensional catalog optimization formulation.

NOMENCLATURE:

$C$ = Expected present value of initial cost
$C_T$ = Expected value of total cost
$c_o$ = constant
$F$ = Objective functional
$f$ = Probability density
$G$ = Importance ratio
$G_i$ = Discrete value of importance ratio
$H$ = Loss or damage function
$L$ = Expected present value of loss function
$P(*)$ = Probability
$q$ = Parameter of the beta distribution
$R$ = (Ramdom) resistance
$\bar{R}$ = Expected value of resistance
$r$ = Parameter of the beta distribution
$S$ = (Ramdom) load effect
$\bar{S}$ = Expected value of load effect
$V$ = Coefficient of variation
$w$ = $1 + V^2$
$W_n$ = Total catalog waste
$w_i$ = Waste function
$x$ = Demand variable
$x_i$ = Supply variable
$Z$ = Normalized total cost
$Z_o$ = Optimum normalized total cost
$\alpha$ = Coefficient in approximate expression for $P$
$\alpha'$ = $\alpha(W_S/W_R)^{\beta/2\sigma}$
$\beta$ = Coefficient in approximate expression for $P$
$\theta$ = Central safety factor
$\theta_o$ = Optimal central safety factor
$\theta_{oi}$ = Discrete value of central safety factor
$\sigma$ = $(\ln W_R W_S)^{1/2}$

INTRODUCTION

The lack of information about the behaviour of structures in service combined with the use of codes containing high safety factors can lead to the view, still held by some engineers, naval architects as well as by many members of the public, that absolute safety can be achieved. Absolute safety is of course unobtainable, and such a goal is also undesirable, since the attempt to achieve absolute safety would consume too many of our finite resources.

Accordingly, it is now widely recognized that some risk of undesirable structural performance must be tolerated. This leads to the debated and open question of how to select structural risk levels in order to obtain optimum structural performance, taking into account the available economical resources and competing demands.

During recent years, a new generation of probability-based design codes have been formulated (1-5). The reliability levels for these codes were selected on the basis of intuition influenced particularly by the current, good or bad, level of performance of existing structures. The load and resistance factors in these codes are functions of the coefficients of variation of the basic random variables, and thus reflect a certain safety level differentiation with respect to uncertainty in the values specified in the code. So-called "Importance Factors" have been introduced as a practical, although intuitive means of achieving *differentiation of safety* levels. Importance factors usually consist of multiplicative values to be applied to the standard design values of the actions or to the specified values of the resistance in order to modify the probability of failure.

A practical rationale for the selection of safety factors for different structural types is essentially *lacking* in the theory of codified design; this is especially true in the context of marine structures. The aim of this paper is to take a closer look at this problem and to propose ways to put its mathematical aspects into a simple formulae.

UTILITY

The purpose of a structural design standard is to regulate the process of production of structures in a way that is optimum for society. If all quantities that a structural standard dealt with were deterministic it would be possible to optimize the standard in principle by listing all possible options in order of preference and select the option ranked on the first place. There is, however, much uncertainty associated with structural behaviour, so in order to optimize, a scalar that reflects society's intensity of preferences should be assigned, and the option associated to the highest value of the scalar chosen. By *definition* this scalar is the utility. When it comes to quantities that can be measured in monetary terms, (these are in general so small for individual choices as compared with society's resources) little error is introduced by assuming that utility is a linear function of money.

The safety parameter of a *structural standard* is *practically* optimum if it maximizes the expected present value of economic benefits, and minimizes that of economic losses for society. For simplicity in this paper, central safety factors and their corresponding safety levels (probabilities of failure) will be the code provisions to optimize.

OBJECTIVE FUNCTION

Any rational economic optimization study demands the assessment of initial costs, C, and potential future damage costs, L, due to structures entering limit states. In the case of marine structures these costs encompass the following component costs:

1. Initial Cost
   a. Design Costs
   b. Material Costs
   c. Construction Costs
   d. Cost of Supervision
   e. Finishing (Construction costs which are not structural)

2. Potential Future Damage Costs
   a. Cost of Investigation
   b. Cost of Strengthening
   c. Damage for Injury or Death
   d. Loss of Revenue
   e. Cost of Pollution
   f. Loss of Cargo
   g. Legal Costs
   h. *Cost of Removal*

Since a linear relation between utility on one hand and the monetary equivalent of benefits to society, costs and losses in the other has been assumed, additivity of utilities follows (6). Thus an appropriate objective function to minimize is:

$$C_T = C + L \qquad (1)$$

in which C and L are the respectively expected present values of initial costs and losses, due to structures entering limit states. Equation (1) implies that changes in benefits derived from the structure's existence due to changes in design, are incorporated into C, L or both.

Initial Cost Function

Let the central safety factor, θ, be defined as the ratio of the expected resistance, $\bar{R}$, and the expected load effect, $\bar{S}$. In conventional ship and offshore structures, little error is induced by assuming that the initial cost (whether an entire structure or a structural element is being considered) at the neighborhood of the optimum is a linear function of the safety factor. Accordingly the expected present value of the initial cost, C, may be expressed as:

$$C = c_o + c_1\theta \qquad (2)$$

where $c_o$ and $c_1$ are constants that depend on the type of structure.

Damage Cost Functions

Actions on marine structures include wave, wind, ice and current loads, imposed deformations, wear and corrosion, etc. They are functions of time, and so is the structural response. *Hence, L should be obtained* by integrating with respect to time. Sometimes however, it is worth idealizing matters as if the structures were subjected to the actions at a fixed time during their lifespans. Now, assume that the response can only take place at a fixed time and that the only mode of structural damage or failure is a single limit state. The loss, H, at the time of failure will be a step function of the central safety factor, say at $\theta = 1.0$, as shown in Figure 1. The expected present value of the loss due to failure or damage involves multiplication by the probability, $P(\theta)$ that the limit state be reached, then:

$$L = HP(\theta) \qquad (3)$$

If there is a series of possible limit states, either independent of each other or in cascade, that is, such that each limit state beyond the first implies that the structure has entered the previous one, as shown in Figure 2, Equation 3 must be replaced with:

$$L = \sum_{i=0}^{n} (H_i - H_{i-1}) P_i \qquad (4)$$

where subscript i identifies the ith limits state and $H_o = 0$. If the limit states are dependent events, the $P_i$'s are conditional probabilities. A combination of independent sequential modes of failure or damage is also possible.

If H varies gradually with $\theta^{-1}$, say the first portion of the curve in Figure 3 the loss function becomes:

$$L = \int_{o}^{\infty} H \left(\frac{dP}{d\theta^{-1}}\right) d\theta^{-1} \qquad (5)$$

under the assumption that each value of $\theta$ can only be attained at a fixed value of time. It is worthwhile to illustrate practical situations of the three cases presented.

Consider first the bottom plating along the forefoot of a ship for which the only significant limit state is collapse. Then yielding of the plate elements or permanent set due to slamming in rough seas may be irrelevant up to a critical state beyond which collapse will take place, then the idealization of Figure 1 is adequate.

Next, consider a slab in a concrete offshore platform. At small loads the slab would develop hair cracks. Their widths are not important so long as they do not exceed about 0.1 mm. In this range there is at most an insignificant loss. At worst, the owner may decide to repaint the slab, in some cases earlier than if the cracks have not appeared. When the load is increased the crack width grows. Beyond approximately 0.1 mm, human reaction is quite unfavorable to the presence of cracks, but their precise widths are not very significant, at least up to 0.25 mm. However, if the slab is unprotected from the outside environment, as is the case in offshore platforms, large cracks may originate corrosion of the reinforcement bars, involving at some stage a more serious loss due to spalling of concrete. If the slab is covered with an impermeable material, large deflections in the slab may cause ponding. At some stage the slab may also vibrate excessively, finally, at higher loads or smaller resistances the slab may collapse. In this case the loss function may look like Figure 2.

In the case of structural systems with large capability of load redistribution following an element failure, the curves increase gradually, since the structure can accommodate overload with little damage costs. The damage function for those structures may look like Figure 3.

## OPTIMAL SAFETY FACTORS

For simplicity only the first case (that of a structural element having a single relevant limit state) will be dealt with in the following. However, the solution procedure is the same for all three cases described.

After substitution of Equations 2 and 3 in Equation 1, the function to be minimized may be rewritten as:

$$C_T = c_o + c_1\theta + HP \qquad (6)$$

or equivalently, in terms of the normalized cost $Z = (C_T - c_o)/c_1$, the following function may be minimized:

$$Z = \theta + GP \qquad (7)$$

where the relation $G = H/c_1$ will be called the <u>importance ratio</u> of the structure and $P = P(R < S)$ is the failure probability.

Now, consider the commonly used approximation concerning the failure probability (7)

$$P = \alpha \exp\left(\frac{-\beta}{\ln(\bar{R}/\bar{S})}\right) \qquad (8)$$

where $\alpha$ and $\beta$ are constants. Equation 8 is a good approximation, valid in the range of relatively small failure probabilities for a wide variety of probability distributions of R and S. If both of these variables have

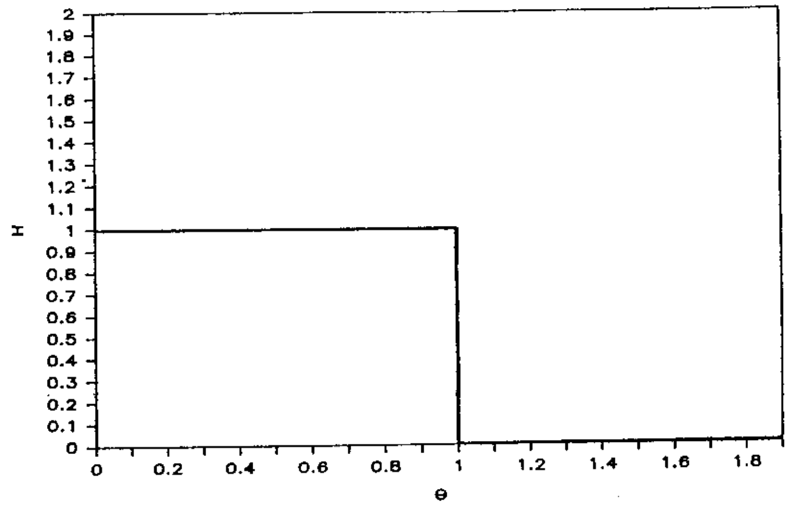


**Figure 1** DAMAGE FUNCTION FOR A SINGLE LIMIT STATE

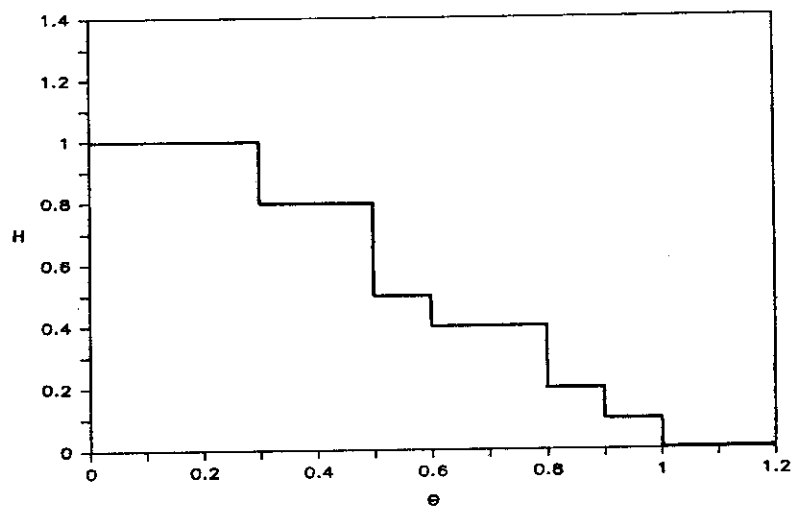


**Figure 2** LIMIT STATES IN CASCADE

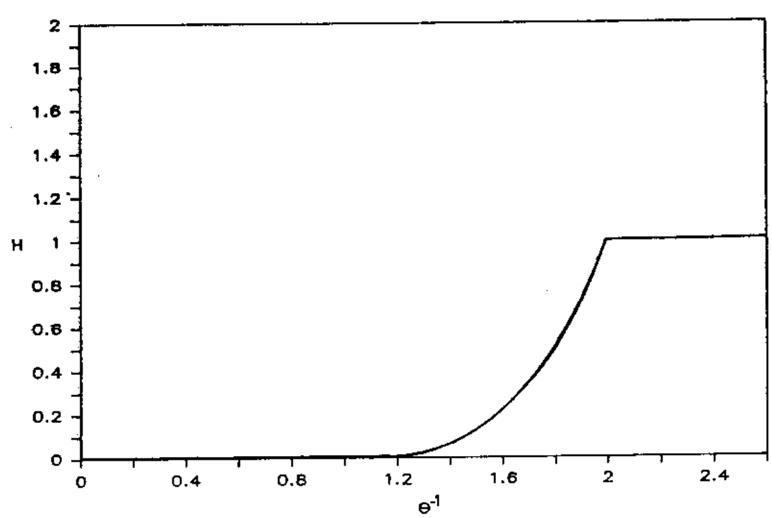


**Figure 3** SMOOTH DAMAGE FUNCTION

lognormal distributions, in the range of P from $10^{-2}$ to about $10^{-6}$ then $\alpha = 460$ and $\beta = 4.3$. If the following variables are defined:

$$\sigma = (\ln W_R W_S)^{1/2},$$

$\alpha' = \alpha\,(W_S/W_R)^{-\beta/2\sigma}$ and $W = 1 + V^2$ (where V means coefficient of variation), Equation 8 may be transformed into:

$$P = \alpha' \theta^{\beta/\sigma} \qquad (9)$$

Equating to zero the derivative of Z (Equation 7) with respect to θ gives the optimal central safety factor, $\theta_o$, as a function of the importance ratio G, then,

$$\theta_o = \left(\frac{G\beta\alpha'}{\sigma}\right)^{\frac{1}{\beta/\sigma+1}} \qquad (10)$$

and the optimal (minimum) normalized cost:

$$Z_o = \left(\frac{G\beta\alpha'}{\sigma}\right) + G\alpha'(G\beta\alpha')^{\frac{\beta}{(\beta+\sigma)}} \qquad (11)$$

Figure 4 shows graphically the principle of the solution. It is observed that normalized total costs increase dramatically on the left side of the minimum but increase only gradually on the safer side. Equations 10 and 11 are shown in Figures 5 and 6 as a function of the coefficients of variation of the load, $V_S$ and the importance ratio G, and for $V_R = 0.2$.

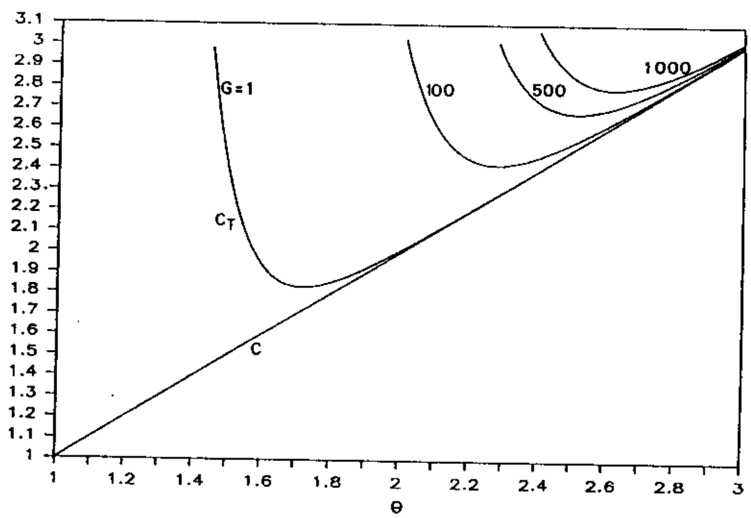


**Figure 4** **INITIAL and TOTAL COSTS**

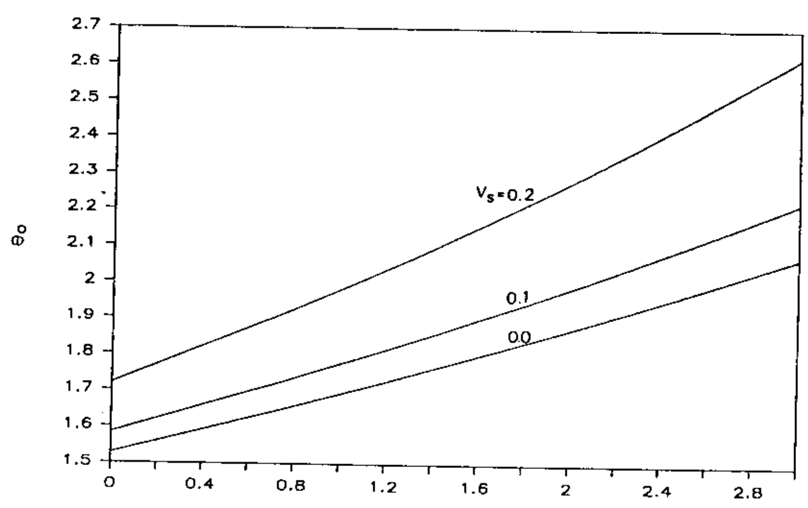


**Figure 5** **OPTIMAL CENTRAL SAFETY FACTORS, $V_R$ : 0.2**

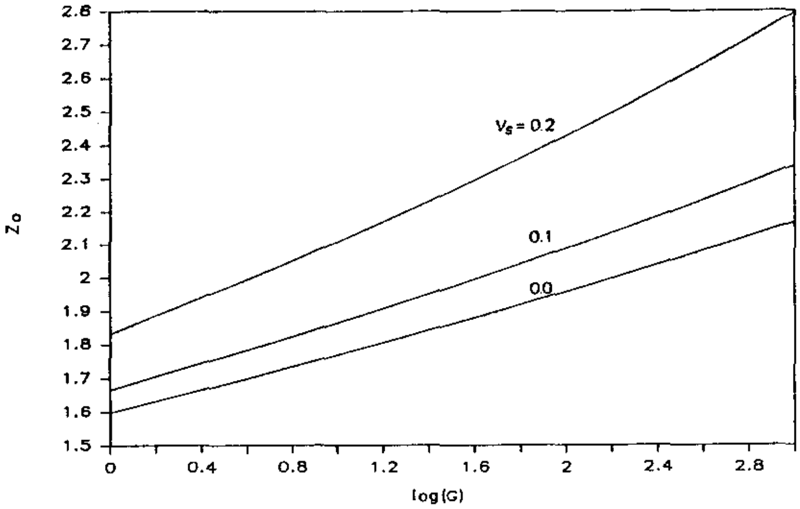


**Figure 6 OPTIMAL NORMALIZED TOTAL COSTS, $V_R$: 0.2**

Generally, G lies between 10 and $10^3$, this range is representative of most marine and ship structures. Much lower values may correspond to *structures whose total cost is very* sensitive to their structural capacity and whose failure does not usually entail consequences far beyond the structure itself, as is the case with navigational buoys. *As we move from G* = 10 to 1000 we may be going through the range of small boats to oil production platforms whose collapse would possibly cause the death of several people or would cause very high levels of pollution of the sea.

It would be unjust for design standards to require exceptionally high reliabilities that such structures should have, but equally objectionable not to include clauses in structural standards that would *force* owners of structures claimed to be exceptionally safe to comply with appropriate requirements concerning their reliability. (Typically one could have standards classify structures *into levels of safety and* contain clauses like "The Naval Architect/Structural Engineer may supply a certificate stating that the structure meets the standard for safety-level III provided its reliability against local collapse under the assumptions specified in section X is not less than $10^{-3}$ and against total collapse for the same assumptions, not less than $10^{-5}$", and make some reference to return periods).

It may be observed in Figure 5 that the optimal central safety factor is very sensitive to G and $V_S$. Design for a fixed safety factor (fixed reliability) can clearly lead far from the optimum. *Even if through an exercise* of engineering judgement one could say how θ should vary as a function of the importance of the structure, one would have to vary θ as a function of $V_S$, and this is not generally recognized in marine design regulations. The larger $V_S$ is, the more expensive it is to attain a given level of safety. The sensitivity increases when the loss in case of failure is an *increasing function* of the load acting at the time of collapse.

For regulatory and code specification purposes and for practical reasons, it *is convenient* to use a discrete set of values of the safety factor. Selection of the values should be based on the frequency of occurrence of each design condition, in this case defined by the variables G, $V_R$ and $V_S$, and on the overall cost to society associated to each safety factor specified. This consideration leads to a catalog problem which is described next.

## OPTIMAL SELECTION OF SAFETY FACTORS

Simplicity is a most desirable feature in design standards. It encourages acceptance of the document and reduces the probability of errors. One way to simplify a standard is by reducing *the number of values that a* given parameter can take. This may be accomplished by grouping structures and structural elements into types, each type with a single value of the code parameter. Using a discrete set of values, though, leads to some increase in the expected initial cost (overdesigned structures), or an increase in the expected present value of the losses due to potential entrance into limit states (underdesigned structures). From the standpoint of this paper, the question *of the optimum set* of values of the safety parameters or safety levels is:

how many and exactly which values should be specified in a design standard of marine structures in order to minimize the expected costs of overdesign and underdesign structures, given the distribution of the demand importance ratio to be expected? Choice of the number of values involves quantifying the cost or utility of rather intangible concepts (e.g. hesitation of designers to use a complicated standard, and the increase on errors due to the high number of values to choose from), and lies beyond the scope of the present paper. However, once the number of values is chosen, though, the problem may be formed under the class of catalog optimization problems.

## Catalog Optimization Problem

The general catalog optimization problem can be stated as: Given a demand vector space x, a probability distribution of demand f(x), and an objective functional F(x,x'); find a set of vectors $x' = x_1, x_2, \ldots, x_n)$, called the supply set or catalog, and a mapping h(x) with range x', called the supply policy, to minimize the expected value F of F(x,h(x)).

The determination of a catalog of safety factors or safety levels may be viewed as a one-dimensional version of the catalog optimization problem. Some solutions to solve this problem have been developed using Calculus (8), Dynamic Programming (9, 10), or graphical methods (9).

Lind (11) has shown that there is a necessary and sufficient condition for the boundaries of the supply sub domains to be optimal when the number, n, of supply elements have been decided upon and the boundaries may be chosen freely. It is assumed that the domain, D, is divided into n sub supply domains, one for each element of supply. Supply subdomains are closed sets and their intersections are called supply boundaries. It is also assumed that to each supply subdomain the corresponding optimum value of the supply variables is assigned. The condition is that the loss or waste function due to discretization be continuous at supply

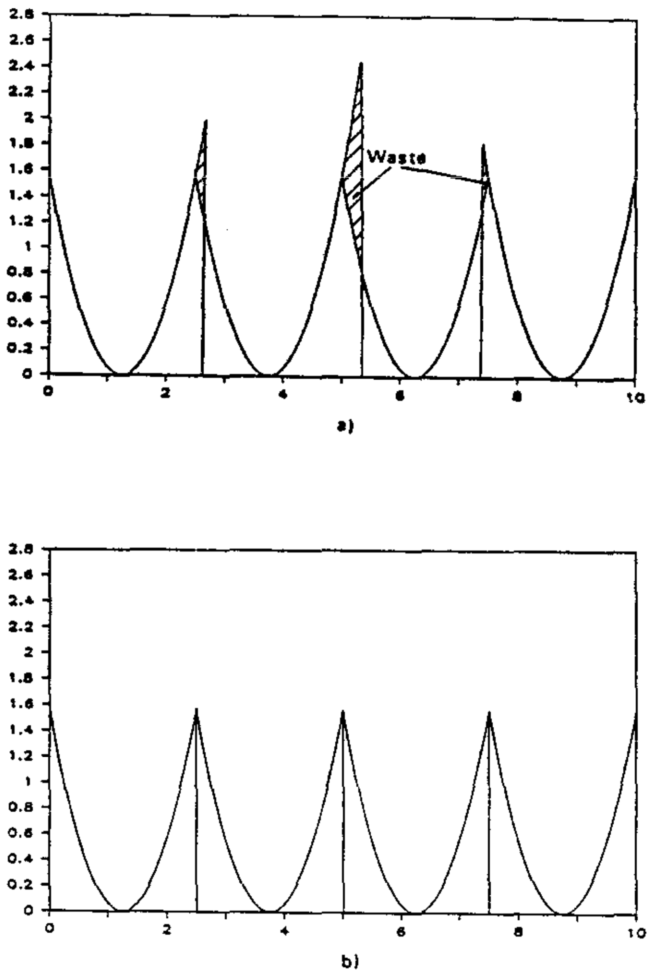


Figure 7 LIND'S OPTIMALITY CONDITION; a) SUBOPTIMAL, b) OPTIMAL

boundaries. A graphical representation of this condition is shown in Figure 7. Next section presents a derivation of a waste function that can be used in the selection of a set of central safety factors.

## Catalog Waste Function

The objective at hand is to find a set of discrete values of the optimal central safety factor that minimizes the total waste due to discretization. The waste due to discretization is the difference between the value $Z(\theta_{oi})$ as is given by Equation 10 when $\theta$ is taken as the constant $\theta_{oi}$ over a set of structures represented by subdomain $A_i$ of the importance ratio G, and the value $Z(\theta_o)$ when $\theta$ is made equal to $\theta_o$ for each structure in the subdomain $A_i$. From now on this subdomain will be called the applicability of central safety factor $\theta_{oi}$. For each structural type the discretization waste may be expressed as:

$$w_i = (Z(\theta_{oi}) - Z(\theta_o))\ f(\theta_o) \quad (12)$$

where $f(\theta_o)$ is the demand frequency for $\theta_o$. As an example consider a specified central safety factor $\theta_{oi} = 2.51$ corresponding to the optimal central safety factor for a structure with G = 500, $V_R = V_S = 0.2$ and let $f(\theta_o) = 1$, then $w_i = 0.679$ when G = 1; $w_i = 0.116$ when $G_i = 100$; and $w_i = 0.052$ when G = 1000. These results are shown in Figure 8.

Adding all individual $w_i$'s over the applicability domain $A_i$ of $\theta_{oi}$ results in the total waste due to the use of $\theta_{oi}$.

$$w_{Ti} = \int_{A_i} (Z(\theta_{oi}) - Z(\theta_o))\ f(\theta_o) \quad (13)$$

and the total catalog waste for a catalog of central safety factors of size n may be computed by:

$$W_n = \sum_{i=1}^{n} \int_{A_i} (Z(\theta_{oi}) - Z(\theta_o))\ f(\theta_o) d\theta_o \quad (14)$$

Now, since $\theta_o$ is a monotonic increasing function of the importance ratio, G, and once the values of $V_R$ and $V_S$ are given (see Equation 10 and Figure 5), the total catalog waste may be alternatively written in terms of G as,

$$W_n = \sum_{i=i}^{n} \int_{A_i} (Z(G_i) - Z(G))\ f(G)dG \quad (15)$$

where Z(G) is the minimum cost of a structural element designed for an importance ratio G and optimal central

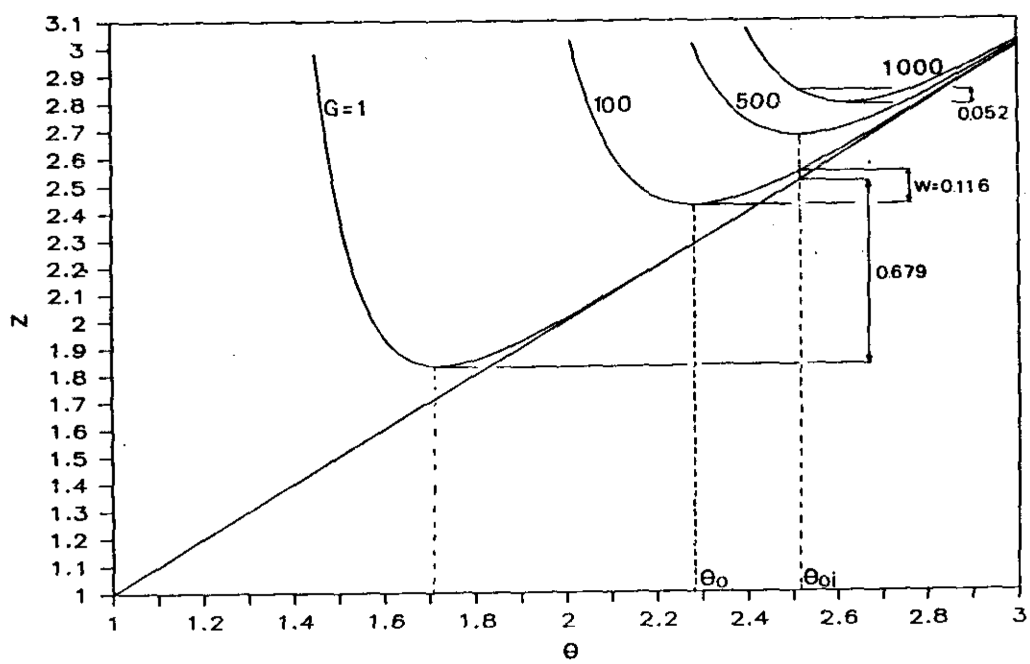


**Figure 8 WASTE DUE TO DISCRETIZATION**

safety factor $\theta_o(G)$, and $Z(G_i)$ is the cost of the same element designed with a specified central safety factor $\theta_{oi}(G_i)$, corresponding to a specified discrete value $G_i$. If the number of required discrete values is fixed, and the applicability domains are computed, the determination of the $G_i$ values that minimize Equation 15 can be done through mathematical programming algorithms. (For a detailed description of the computation of applicabilities and the applicaton of the algorithms the reader is referred to Ferregut, (1986)).

As an example consider a uniform distributed demand of importance ratios defined by the region $1 \leq G \leq 1000$, and let the design conditions defined by all structures and structural elements with $V_R = V_S = 0.2$. If n is taken as three, the values of $G_i$ that minimize Equation 15 are $G_1 = 57.3$, $G_2 = 267.3$ and $G_3 = 710.5$ with corresponding optimal central safety factors $\theta_{o1} = 2.20$, $\theta_{o2} = 2.42$ and $\theta_{o3} = 2.57$, and total catalog waste $W_3 = 9.43$. Figure 9 shows the continuous and discrete values of the demanded central safety factors. If instead of a uniform demand a beta distributed demand is used

$$f(G) = \frac{\Gamma(q+r)}{\Gamma(q)\Gamma(r)} \frac{(G-a)^{q-1}(b-G)^{r-1}}{(b-a)^{q+r}}$$

with parameters a = 1, b = 1000, q = 2 and r = 4 the following results are obtained: $G_1 = 95.5$, $G_2 = 248.3$, $G_3 = 501.1$ with corresponding optimal central safety factors: $\theta_{o1} = 2.27$, $\theta_{o2} = 2.41$, $\theta_{o3} = 2.51$ and total catalog waste $W_3 = 5.56$. These results are also shown in Figure 9. Notice that the central safety factors computed assuming a beta distributed demand vary at most 3.1% from the values computed using a uniform demand. Nevertheless, the applicability of $\theta_{o2}$ narrows at the same time that the applicability of $\theta_{o3}$ expands by a considerable length. This is due to the skewness of the right-hand side of the beta distribution. This result shows that the total waste due to the use of a discrete set of values of safety factors and the safety factors themselves are highly dependent on the choice of the demand distribution.

CONCLUSIONS

When establishing optimal allowable safety levels for structural design, several different sets of conditions have to be acknowledged and treated differently. For example, when it is likely to assume that the structure or structural element enters only a single limit state or never fails, it is possible to use charts such as Figure 4 after converting central safety factors into failure probabilities.

It has been shown that optimal levels of safety depend on the value taken by the importance ratio as well as on the values of the coefficient of variation on loads and resistances. These conditions are not currently recognized in design rules for marine structures. Optimal levels of safety also depend on the choice of the loss

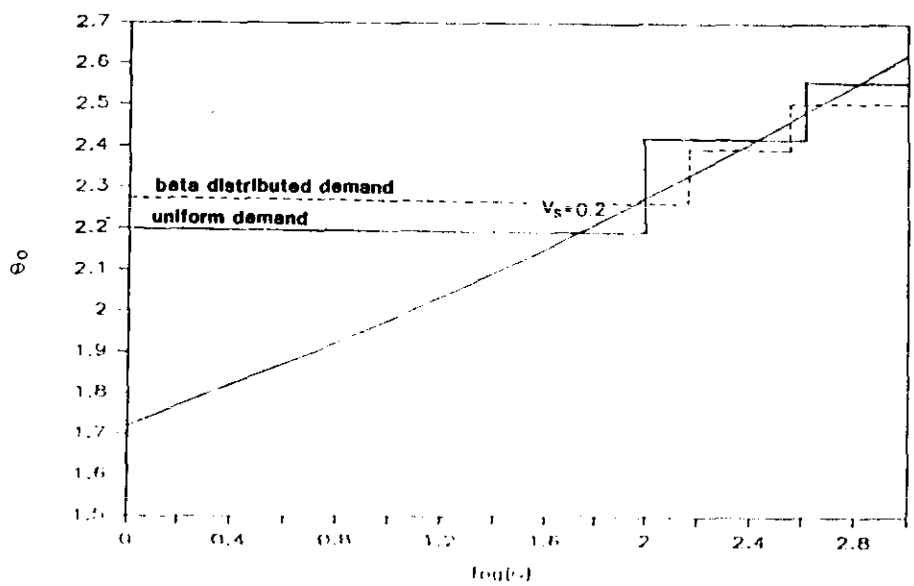


Figure 9 CONTINUOUS and DISCRETE VALUES of $\theta_o$

function. Since loss functions are ill defined for most problems of practical interest a possible area for further research would be the study of suitable loss functions for marine structures.

If a discrete set of values of the central safety factors is required for a design standard, they can be obtained through the solution of a catalog problem.

This paper explores a hitherto neglected subject in structural design. The subject is very broad and with many facets that will require special studies and some new techniques. The author hopes this work will encourage further research in this direction.

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