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Overall Structural Response of a Ship Struck in a Collision

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It is shown that the vibration analysis can be efficiently employed to predict the elastic and eventual nonlinear response - or even collapse - of the whole ship when struck in a collision. Based on the assumption of an impact force-time law , e.g. taken from model tests, a procedure leading to a time-dependent representation of bending moment, shear force and state of stress is developed. An illustrative example of application to the design of a nuclear-powered container ship, developed by GKSS Forschungszentrum Geesthacht in the FR of Germany, is described. It shows that high accelerations and collapse of the whole ship may be caused by a collision with a ship having a strong bow construction.

NOMENCLATURE

A = cross section area of the struck ship
$B = EI_z$ = bending rigidity of struck ship for bending in horizontal plane
e = distance between striking force and center of gravity
E = Young's modulus
i_G = radius of gyration of the struck ship
I_z = moment of inertia
L = length of struck ship
m_1 = mass of striking ship
$m_2, \mu = m_2/L$ = mass of struck ship including hydrodynamic mass, mass per unit length
$M_z(x,t)$ = bending moment in horizontal plane
$P(t), P_o$ = impact force, maximum force
$q(\tau)$ = elemental impulse of impact force $P(\tau)$
$Q(x,t)$ = shear force
$\bar{t}, t, t^*, \tau$ = time (see fig.3)
T = half period of Fourier expansion
$T_j(t)$ = j-th time function of elastic vibration
$U(t)$ = strain energy
v_1 = velocity of striking ship
$w(x,t)$ = bending deflection of the ship's axis
x, x_G, x_o = distance between bow and point of reference, center of gravity, shock force resp.
$X_j(x)$ = j-th eigenfunction of elastic vibration in horizontal plane
$y(x,t)$ = deflection of the ship's axis
$y_G(t)$ = deflection of center of gravity
λ_j = j-th eigenvalue of eigenfunction X_j
σ_y = yield stress
$\xi_i(x)$ = reduced eigenfunction X_i (see eq.(19))
ϕ = angle of undeformed ship's axis relative to x-axis
ω_j = j-th angular frequency of j-th eigenfunction X_j

1. INTRODUCTION

The existing literature includes more than one hundred papers on collision problems of ships, as the latest reviews /1/ and /2/ show. Nearly all of these papers concern the local structural response of a striking and struck ship in order to get force-penetration characteristics and to provide a basis for recommendations regarding designs for protection against collision penetration. Although many of these papers use impact dynamics for the colliding ships to develop the balance of energy, there is, to the author's knowledge, no paper which uses vibration analysis to investigate thoroughly the elastic and eventual nonlinear response - or even collapse - of the whole ship as a result of a collision impact.

Although a strong side construction may be advantageous for protecting holds with dangerous goods or reactor rooms of nuclear powered ships, the resulting high collision forces and relatively short collision durations may induce high stresses and accelerations. In the worst case, the collision forces may induce the collapse of the struck ship hull girder even though the protected compartmants may not have been destroyed by the impact force. To the author's knowledge, this aspect has not been considered up to now. Although papers /3/ to /6/ investigate the collision dynamics of colliding ships in more detail than the other papers they do not do so in the sense desribed above. Therefore, the present paper intends to fill this gap. Several publications, especially from the experimental field, show that the approximate force-time dependencies of a collision process may be assumed as known. These dependencies of a collision process may be expanded into a Fourier series and taken as input for a vibration analysis of the struck ship. For simplicity the ship is modelled as an elastic beam with constant moment of inertia. With these assumptions, kinematics and stress-time functions can be computed approximately for the transient state immediately after impact. This method has been applied to the design of a nuclear-powered container ship with a strong protective side construction, developed recently in the FR of Germany by WOISIN /7/ of GKSS. It shows that high accelerations and possibly collapse of the whole ship may be caused by a collision with a ship having a strong bow construction.

2. EQUATIONS OF MOTION

The equations of motion are derived with the following assumptions:

- The striking ship collides with a known force P(t) and with its axis in a line perpendicular to the axis of the struck ship at a distance e from its center of gravity G (see fig.1),
- the hydrodynamic forces are taken into account by additional masses,
- influences of rotary inertia and shear deformation are not included in the vibration analysis,
- the struck ship is modelled as an elastic beam with constant moment of inertia.

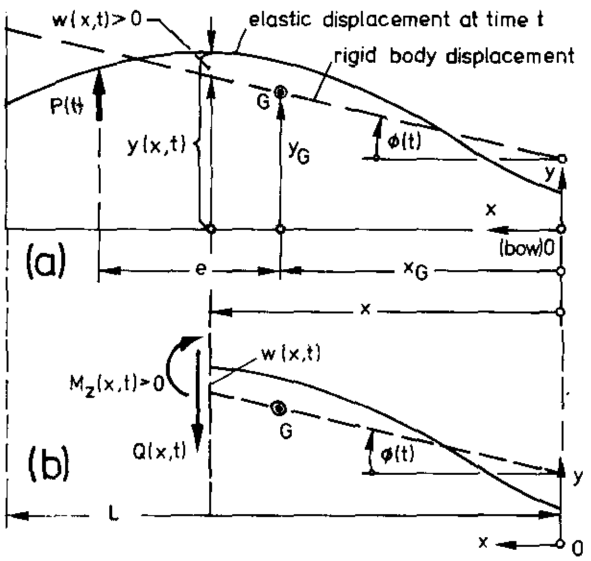

Fig.1 (a) Displacement of the ship's axis
(b) Bending moment M_z and shear force Q

Fig.1 shows the deflection curve at time t in an (x,y) coordinate system coinciding with the position of the struck ship at the beginning of the impact (t=0). The dotted line indicates the struck ship's axis at time t when its deformation is not considered (rigid-body motion) and the bending deflection is w(x,t). Then, for $\phi(t)<<1$

$$y(x,t) = y_G(t) + (x - x_G)\phi(t) + w(x,t). \quad (1)$$

This gives the absolute acceleration of the struck ship's axis

$$\frac{\partial^2 y}{\partial t^2} = \ddot{y}_G(t) + (x - x_G)\ddot{\phi}(t) + \frac{\partial^2 w}{\partial t^2} \quad , \quad (2)$$

$\ddot{}$ meaning the second order derivative with respect to t. Then, the general differential equation for the lateral elastic vibration of the struck ship in the horizontal plane - for simplicity reduced to a bar with constant bending rigidity $B = EI_z$ - is

$$B\cdot\frac{\partial^4 y}{\partial x^4} = B\cdot\frac{\partial^4 w}{\partial x^4} = -\mu\frac{\partial^2 y}{\partial t^2}$$
$$= -\mu\cdot\left[\ddot{y}_G(t) + (x - x_G)\ddot{\phi}(t) + \frac{\partial^2 w}{\partial t^2}\right] \quad .(3)$$

By the laws of linear and angular momentum for the struck ship

$$\ddot{y}_G(t) = \frac{1}{m_2}P(t) \text{ and } \ddot{\phi}(t) = \frac{e}{m_2 i_G^2}P(t), \quad (4)$$

m_2 being the mass of the struck ship including the added hydrodynamic mass, i_G the radius of gyration, μ the mass of the struck ship per unit length and P(t) the impact force. Therefore, the motion induced by the impact force can be expressed by

$$\mu\cdot\frac{\partial^2 w}{\partial t^2} + B\cdot\frac{\partial^4 w}{\partial x^4} = \frac{\mu}{m_2}P(t)\left[1 + (x - x_G)\frac{e}{i_G^2}\right]. \quad (5)$$

3. EIGENFUNCTIONS AND EIGENVALUES OF THE FREE MOTION

Firstly, it is necessary to determine the solution of the homogeneous eqation (5), setting P(t)=0. By the approximate separation process

$$w(x,t) = \sum_{j=1}^{J} X_j(x)T_j(t) \quad (6)$$

one obtains from eq.(5) the following system of 2J ordinary differential eqations

$$\ddot{T}_j(t) + \omega_j^2 T_j(t) = 0 \quad , \quad j=1,\ldots J \quad (7)$$

$$X_j''''(x) - \left(\frac{\lambda_j}{L}\right)^4 X_j(x) = 0 \quad (8)$$

with the square of the angular freqency ω_j

$$\omega_j^2 = \left(\frac{\lambda_j}{L}\right)^4 \frac{B}{\mu} \quad , \quad (9)$$

λ_j being the eigenvalue of the eigenfunction X_j and "" the 4-th order derivative with respect to x. After adjusting the integration of eq. (8) to the boundary conditions of vanishing moments and shear forces at the ends of the struck ship,

$$X_j''(0) = 0 \quad , \quad X_j''(L) = 0$$
$$X_j'''(0) = 0 \quad , \quad X_j'''(L) = 0 \qquad (10)$$

the normalized eigenfunctions are obtained

$$X_j(x) = \sin\frac{\lambda_j x}{L} + \sinh\frac{\lambda_j x}{L} + F_j(\lambda_j)\left[\cos\frac{\lambda_j x}{L} + \cosh\frac{\lambda_j x}{L}\right] \qquad (11)$$

with $F_j(\lambda_j) = \dfrac{\cos\lambda_j - \cosh\lambda_j}{\sin\lambda_j + \sinh\lambda_j} \approx -1$.

The transzendental equations for determination of λ_j

$$\cos\lambda_j \cdot \cosh\lambda_j = 0 \qquad (12)$$

deliver the eigenvalues

$$\lambda_j \approx (2j+1)\frac{\pi}{2} \quad . \qquad (13)$$

4. COMPLETE SOLUTION FOR THE FORCED RELATIVE MOTION w(x,t)

Instead of solving eq.(5) for the total motion it is more appropriate to use d'Alembert's principle for the relative motion w(x,t), according to which the virtual work of inertia forces, impact forces and elastic forces must vanish. The inertia forces per unit length $dx = dm/\rho A$ of the relative motion w(x,t) result from eq.(6)

$$-\rho A \cdot \frac{\partial^2 w}{\partial t^2} = -\mu \sum_{j=1}^{J} X_j(x)\ddot{T}_j(t) \quad . \qquad (14)$$

With the virtual displacement $\delta w = \delta T_i(t) X_i(x)$, and taking account of the orthogonality relations for the eigenfunctions we obtain the virtual work of the inertia forces

$$-\mu \int_{x=o}^{L} \frac{\partial^2 w}{\partial t^2}\delta w dx =$$
$$= -\mu \sum_{j=1}^{L} \ddot{T}_j(t)\delta T_i(t) \int_{x=o}^{L} X_j(x) X_i(x) dx =$$
$$= -\mu \ddot{T}_i(t)\delta T_i(t) \int_{x=o}^{L} X_i^2(x) dx \quad . \qquad (15)$$

With the strain energy of the ship reduced to a bending beam with constant bending stiffness B

$$U(t) = \frac{1}{2}B \int_{x=o}^{L} \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx =$$
$$= \frac{1}{2}B \int_{x=o}^{L} \sum_{j=1}^{J} X_j''^2(x) T_j^2(t) dx$$

the virtual work of the elastic forces is given by

$$-\delta U = -\frac{\partial U}{\partial T_i}\delta T_i =$$
$$= -BT_i(t)\delta T_i(t) \int_{x=o}^{L} X_i''^2(x) dx \quad . \qquad (16)$$

Finally, the impulse force P(t) acting at the point x_o with the virtual displacement

$$\delta w|_{x_o} = \delta T_i(t) X_i(x_o)$$

performs the virtual work

$$P(t)\delta T_i(t) X_i(x_o) \quad . \qquad (17)$$

By summing up the terms (15) to (17), equating to zero, dividing by δT_i, and referring to eq.(7),

$$\ddot{T}_i(t) + \omega_i^2 T_i(t) = P(t)\xi_i(x_o) \qquad (18)$$

is obtained with

$$\omega_i^2 = \frac{B}{\mu} \frac{\int_{x=o}^{L} X_i''^2(x) dx}{\int_{x=o}^{L} X_i^2(x) dx}$$

and

$$\xi_i(x_o) = \frac{X_i(x_o)}{\mu \int_{x=o}^{L} X_i^2(x) dx} \quad , \qquad (19)$$

ω_i again being the angular frequency of the i-th eigenfunction of the free motion. By integration in the denominator of eq.(19), with $X_i(x)$ according to eq.(11), it follows approximately

$$\int_{x=o}^{L} X_i^2(x) \approx L$$

so that:

$$\xi_i(x_o) \approx \frac{X_i(x_o)}{\mu L} \quad . \qquad (20)$$

For studying the transient state of the motion caused by a non-periodic force P(t), it is appropiate to use an impulse analysis. It is necessary to determine the effect of all elemental momentum values $q_i(\tau)d\tau = P(\tau)\xi_i d\tau$ per unit mass on the system at the time t (see fig.2). The velocity increase is

$$\frac{d}{d\tau}\dot{T}_i(\tau) = q_i(\tau), \text{ i.e. } d\dot{T}_i(\tau) = q_i(\tau)d\tau .$$

The displacement at the instant t corresponding to the initial velocity $d\dot{T}_i(\tau)$ imparted to the system at the

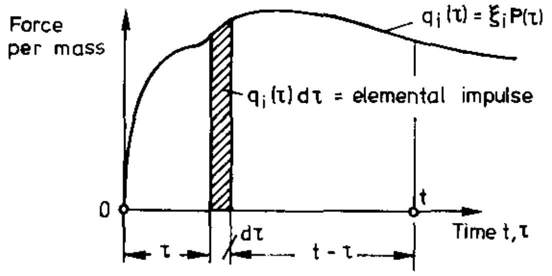

Fig.2 Impulse analysis for w(x,t)

time τ is analogous to the response of a linear undamped system for which

$$\ddot{x}(t) + \omega^2 x(t) = 0 \quad .$$

This holds with the solution adjusted to the initial conditions $x(0) = 0$ and $x(0) = v_o$

$$x(t) = \frac{v_o}{\omega} \sin\omega t \quad .$$

By substituting dT_i for $x(t)$, $q_i(\tau)d\tau$ for v_o, ω_i for ω and $t-\tau$ for t, one obtains as response of the system at the instant t to the impulse $q_i(\tau)d\tau$ at the time τ

$$dT_i(t-\tau) = \frac{q_i(\tau)d\tau}{\omega_i} \sin\omega_i(t-\tau) \quad . \qquad (21)$$

A particular solution of eq.(18) is obtained by integrating the dT_i between 0 and t. Adding the solution of the homogeneous eq.(18), the total solution results as

$$T_i(t) = A_i \cos\omega_i t + B_i \sin\omega_i t + \frac{\xi_i(x_o)}{\omega_i} \int_{\tau=o}^{t} P(\tau)\sin\omega_i(t-\tau)d\tau . \qquad (22)$$

The initial conditions $w(x,0) = 0$ and $\dot{w}(x,0) = 0$ deliver all constants $A_i = 0$ and $B_i = 0$ on account of eq.(6) so that the complete solution of the forced relative elastic motion is

$$w(x,t) = \sum_{i=1}^{I} \frac{\xi_i(x_o)}{\omega} X_i(x) \int_{\tau=o}^{t} P(\tau)\sin\omega_i(t-\tau)d\tau . \qquad (23)$$

5. SOLUTION FOR TWO FORCE FUNCTIONS P(t)

5.1. P(t) Increasing Linearly with t

For the first shock phase the overall response of the struck ship is elastic (regardless of possible large plastic deformation of the ship's structure in the vicinity of the point of impact) and it is suitable to assume a force growing linearly with the time τ (see fig. 3a)

$$P(\tau) = \frac{P_o}{\bar{t}}\tau = c\tau \text{ for the time } 0 \le \tau \le \bar{t} \quad (24)$$

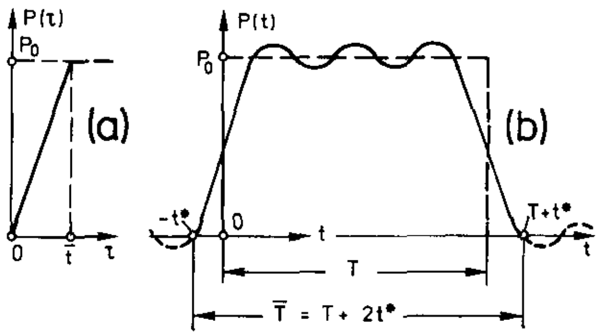

Fig.3 Shock-force time dependency represented by
(a) a linear law
(b) a Fourier series with N = 5

so that eq.(23) after integration yields

$$w(x,t) = c \sum_{i=1}^{I} \frac{\xi_i(x_o)}{\omega_i^3} X_i(x)\{\omega_i t - \sin\omega_i t\} . \qquad (25)$$

It is necessary to add the rigid body motion from eq.(4)

$$\ddot{y}_G = \frac{c}{m_2} t \quad \text{and} \quad \ddot{\phi} = \frac{ce}{m_2 i_G^2} t \qquad (26)$$

which after integration, gives

$$y_G(t) = \frac{c}{6m_2} t^3 + \dot{y}_{Go} t + y_{Go} \quad \text{and}$$

$$\phi(t) = \frac{ce}{m_2 i_G^2} \frac{t^3}{6} + \dot{\phi}_o t + \phi_o \quad . \qquad (27)$$

With the initial conditions

$$y_{Go} = 0, \quad \dot{y}_{Go} = 0, \quad \phi_o = 0, \quad \dot{\phi}_o = 0$$

we obtain from eq.(1), with the approximation (20) for $\xi_i(x_o)$ and with eqs.(25) and (27), the absolute displacement of the ship's axis

$$y(x,t) \approx \frac{c}{\mu L}\left\{\frac{t^3}{6}\{1 + (x - x_G)\frac{e}{i_G^2}\} + \sum_{i=1}^{I} \frac{X_i(x_o)X_i(x)}{\omega_i^3}\{\omega_i t - \sin\omega_i t\}\right\} \qquad (28)$$

and its absolute acceleration

$$\frac{\partial^2 y}{\partial t^2} \approx \frac{c}{\mu L}\left\{t\{1 + (x - x_G)\frac{e}{i_G^2}\} + \sum_{i=1}^{I} \frac{X_i(x_o)X_i(x)}{\omega_i} \sin\omega_i t\right\} \quad . \qquad (29)$$

5.2. Expansion of P(t) into a Fourier Series

For desribing the whole shock process, it is desirable to represent the impact force-time dependency by the Fourier series

$$P(t) = \frac{P_o}{2} + \sum_{n=1}^{\infty}\left\{a_n \cos\frac{n\pi t}{T} + b_n \sin\frac{n\pi t}{T}\right\} \quad . \quad (30)$$

Taking all terms of this series, the force is zero when $t \leq 0$ and $t \geq T$, jumping at $t=0$ to a constant value P_o and switching off at $t=T$ (dotted curve in fig.3b). However, with the finite number N of terms one can better represent the real shape of a shock force curve as obtained by tests. Fig.3b shows an impact force $P(t)$ with $N=5$.

One can even choose the number N so that any desired gradient of the $P(t)$-curve, selected from experimental evidence and corresponding to the slope of the $P(t)$-curve of fig.3a, can be obtained (greater values of N give steeper gradients of the force curve). It is assumed that waviness of the force curve around the constant force P_o is not essential for the shock process. However, the assumption of approximately constant shock forces after a first linear phase is rather realistic, as tests have shown. The Fourier coefficients in eq.(30) are

$$a_n = 0 \quad , \quad b_n = -\frac{P_o}{n\pi}\{\cos n\pi - 1\} \qquad (31)$$

so that we have

$$P(t) = P_o\left\{\frac{1}{2} + \sum_{n=1}^{N}\frac{1-\cos n\pi}{n\pi}\sin\frac{n\pi t}{T}\right\} \quad . \quad (32)$$

Notice that with a finite number N of terms the impulse begins at $t=-t^*$, so that the shock duration is $\bar{T}=T+2t^*$. Inserting eq.(32) into eq.(23), the relative displacement of the ship's axis is obtained at time t and point x, caused by an impact force $P(t)$ at point x_o, as

$$w(x,t) = \sum_{i=1}^{I}\frac{\xi_i(x_o)}{\omega_i}X_i(x)P_o\int_{\tau=-t^*}^{t}\left\{\frac{1}{2} + \sum_{n=1,3..}^{N}\frac{1-\cos n\pi}{n\pi}\sin\frac{n\pi t}{T}\right\}\sin\omega_i(t-\tau)d\tau, \quad (33)$$

the terms in the second series for $n=2,4..$ being zero. The total absolute displacement is determined as in 5.1 by adding the rigid body motion terms.

The Fourier analysis should be restricted to the constant shock force phase, whereas for the first linear phase it is better to apply eq.(25) because that equation does not contain disturbances for $t<-t^*$ as the Fourier series with finite N does (see fig.3b).

6. BENDING MOMENTS AND SHEAR FORCES

Bending moments and shear forces (see fig.1b) for the different load assumptions are derived from eq.(25) and eq.(33) as follows:

6.1. For Linearly Increasing P(t)

The bending moment is obtained from eq.(25) with ω_i from eq.(9), ξ_i from eq.(20) and with the eigenfunctions $X_i(x)$ from eq.(11) as

$$M_z(x,t) = B\frac{\partial^2 w}{\partial x^2} = cL^3\sqrt{\frac{\mu}{B}}\sum_{i=1}^{I}\frac{X_i(x_o)}{\lambda_i^4}\left\{\sinh\frac{\lambda_i x}{L} - \sin\frac{\lambda_i x}{L} + F_i(\lambda_i)\{\cosh\frac{\lambda_i x}{L} - \cos\frac{\lambda_i x}{L}\}(\omega_i t - \sin\omega_i t)\right\} \quad (34)$$

and the shear force is

$$Q(x,t) = B\frac{\partial^3 w}{\partial x^3} = cL^2\sqrt{\frac{\mu}{B}}\sum_{i=1}^{I}\frac{X_i(x_o)}{\lambda_i^4}\left\{\cosh\frac{\lambda_i x}{L} - \cos\frac{\lambda_i x}{L} + F_i(\lambda_i)\{\sinh\frac{\lambda_i x}{L} + \sin\frac{\lambda_i x}{L}\}(\omega_i t - \sin\omega_i t)\right\}, \quad (35)$$

6.2. For a Force Curve Expanded Into a Fourier Series

The bending deflection is obtained by writing the first two terms (N=3) of the second series in eq.(33) as

$$w(x,t) = \sum_{i=1}^{I}\frac{\xi_i(x_o)}{\omega_i}X_i(x)P_o\int_{\tau=-t^*}^{t}\left\{\frac{1}{2} + \frac{2}{\pi}\sin\frac{\pi\tau}{T} + \frac{2}{3\pi}\sin\frac{3\pi\tau}{T} + ..\right\}\sin\omega_i(t-\tau)d\tau \quad .$$

After integration of the first two terms in the bracket, one obtains

$$\begin{aligned} w(x,t) = \sum_{i=1}^{I}\frac{\xi_i(x_o)}{\omega_i}X_i(x)P_o\Bigg[&\frac{1}{2\omega_i}\{1 + \sin\omega_i t\sin\omega_i t^* - \cos\omega_i t\cos\omega_i t^*\} - \\ &- \frac{1}{\pi}\sin\omega_i t\left\{\frac{1}{\pi/T+\omega_i}\{\cos(\tfrac{\pi}{T}+\omega_i)t - \cos(\tfrac{\pi}{T}+\omega_i)t^*\} + \frac{1}{\pi/T-\omega_i}\{\cos(\tfrac{\pi}{T}-\omega_i)t - \cos(\tfrac{\pi}{T}-\omega_i)t^*\} + ..\right\} - \\ &- \frac{1}{\pi}\cos\omega_i t\left\{\frac{1}{\pi/T-\omega_i}\{\sin(\tfrac{\pi}{T}-\omega_i)t + \sin(\tfrac{\pi}{T}-\omega_i)t^*\} - \frac{1}{\pi/T+\omega_i}\{\sin(\tfrac{\pi}{T}+\omega_i)t + \sin(\tfrac{\pi}{T}+\omega_i)t^*\} \pm ..\right\}\Bigg] \end{aligned} \quad (36)$$

and so forth. Then, with the bracket relation $[\cdots]_i = B_{iN}$, the bending moment for N terms of the Fourier series will be

$$M_z(x,t) = \frac{P_o}{L}\sqrt{\frac{B}{\mu}}\sum_{i=1}^{I} X_i(x_o)\Big\{\sinh\frac{\lambda_i x}{L} - \sin\frac{\lambda_i x}{L} + + F_i(\lambda_i)\{\cosh\frac{\lambda_i x}{L} - \cos\frac{\lambda_i x}{L}\}\Big\} B_{iN} \qquad (37)$$

and the shear force will be

$$Q(x,t) = \frac{P_o}{L^2}\sqrt{\frac{B}{\mu}}\sum_{i=1}^{I} X_i(x_o)\lambda_i\Big\{\cosh\frac{\lambda_i x}{L} - \cos\frac{\lambda_i x}{L} + + F_i(\lambda_i)\{\sinh\frac{\lambda_i x}{L} + \sin\frac{\lambda_i x}{L}\}\Big\} B_{iN}\,. \qquad (38)$$

With eqs.(34) and (35) or (37) and (38) the basic equations are in hand for the stress-time analysis below.

7. STRESS ANALYSIS

The stress analysis can be performed with the basic eqations of section 6 by application of conventional methods of the theory of strength of structures. The determination of bending stresses from M_z and shearing stresses from Q is elementary. As Q is mainly transferred through the double bottom and not through the shear center, one has to take account of a torsional moment with additional shear stresses and longitudinal stresses on account of restrained warping; the calculations for this purpose are also well known in engineering. To these stresses one must add the stresses resulting from service conditions and determine the equivalent uniaxial stresses in order to be in a position to judge the danger of collapse of the whole ship. To this end, it is necessary to take into account the increase in yield stress with strain rate and of buckling caused by normal and shear stresses as described in /8/. It is not the purpose of this paper to dwell on details of this stress analysis because the procedure is self evident. Only the results will be discussed in the following section.

8. THE OVERALL STRUCTURAL RESPONSE OF A NUCLEAR POWERED CONTAINERSHIP TO A COLLISION IMPACT

As an example for applying the foregoing theoretical results a ship of interest would be one which can endure a rather high impact force without suffering to much local failure. The design of the nuclear-powered containership described in /7/ seems to be well suited for this purpose because ample results from collision tests are available. The tests were made with models having scale ratios of 7.5 and 12 using several bow constructions and several versions of the protective construction around the reactor-room, as reported in /7/.

8.1. Input Data for the Impact Calculation

The values of input quantities for the dynamic analysis were as in Table I. As point x_o of the force application the middle of the reactor-room was chosen. This is the best protected part of the struck ship where the maximum force of 400 MN can be carried without much local permanent destruction as model tests showed /7/.

The impact force was taken as the maximum value calculated from acceleration measurements in the referred collision tests of /7/, multiplied by the square of the model scale. Assuming this force to be approximately constant during the total impact duration,and postulating an entirely plastic impact, it can be concluded from the basic laws of dynamics that the duration of the impact will be

$$\overline{T} = \frac{m_2 v_1}{\{1 + (\frac{e}{i_G})^2 + m_2/m_1\} P_o}\,, \qquad (39)$$

m_1 being the mass of the striking ship. Assuming $m_2/m_1 = 1.1$, with $e = x_o - L/2 = 40$ m, the total duration of the impact is $\overline{T} = 0.88$ sec.

8.2. Acceleration and Displacement of the Ship's Axis in the First Impact Phase

For this analysis, first of all, the angular frequencies ω_i (1/sec) of the

Table I. General characteristics of the containership

Weight of struck ship		556	MN (55,780 tons)
Mass of struck ship including 50% added hydrodynamic mass	m_2 =	85	$MNsec^2/m$ (2,600 $tonssec^2/ft$)
Mass per unit length	μ =	0.297	$MNsec^2/m^2$ (2.77 $tonssec^2/sqft$)
Length of struck ship	L =	286	m (938.3 ft)
Beam of struck ship		32.3	m (106 ft)
Radius of gyration	i_G =	71.5	m (234.6 ft)
Bending rigidity	B =	1.236	MNm^2 (1,335 tonssqft)
Impact: Velocity of striking ship	v_1 =	10	m/sec (19.42 knots)
Constant maximum force	P_o =	400	MN (40,120 tons)
Force acts at	x_o =	183	m (=0.64L = 600.5 ft)

Table II Angular frequencies ω_i(1/sec) of free motion

i	1	2	3	4	5	6	7	8
ω_i	5.58	15.15	30.14	49.82	74.43	103.96	138.40	177.77

free motion were determined from eqs. (9) and (13) in table II.

For the design of the nuclear facilities, determination of the maximum possible acceleration of the reactor is of primary interest. To calculate this, it is appropriate to use eq.(29) with an impact force $P(\tau) = c\tau$ linearly increasing from zero to $\tau = \bar{t}$, and acting at the point $x_o = 0.64L$ of the reactor. Eq.(39) establishes the duration of the whole impact (for a certain impact scenario) but statements about the duration $\bar{t}$ of the first collision phase can be made only from experimental observations. In all German collision tests /7/, the collision force ascended nearly linearly within the time $\bar{t} > 0.1 \cdot T$ up to its maximum value.

Before one can determine the accelerations, it is necessary to evaluate the relative displacement $w(x,\bar{t})$ of the ship's axis, given by the series in eq.(28), which may be of interest for the stress analysis. Because ω_i is raised to the 3rd power in the denominator of the series elements, I=2 terms give acceptable approximations, i.e. the displacement is determined mainly by the first two eigenfunctions. Fig.4 shows the total displacement $y(x,\bar{t})$, according

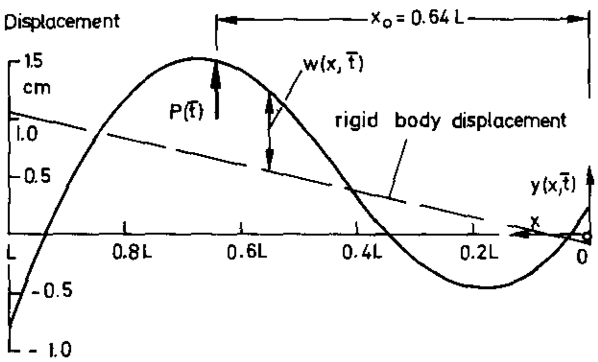

Fig.4 Total displacement $y(x,\bar{t})$ of the ship's axis and elastic response $w(x,\bar{t})$ caused by an impact force P growing linearly within $\bar{t} = 0.1$ sec to its maximum value $P_o = 400$ MN

to eq.(28), and the relative displacement $w(x,\bar{t})$, for the special case when the maximum force $P_o = 400$ MN is reached within the time $\bar{t} = 0.1$ sec. Here it should be mentioned that the stresses resulting from $w(x,\bar{t})$ in this first phase are not very high. For stress analysis the results of section 8.3 are more important, but the accelerations may have their greatest values in the first phase.

In order to determine the acceleration of the reactor, eq.(29) is evaluated for the duration $0 < \bar{t} < 0.18$ sec., and for the point $x = x_o$, so that one obtains

$$\frac{\partial^2 y}{\partial t^2} = \frac{c}{m_2}\left[\bar{t}\{1+(\frac{e}{i_G})^2\} + \sum_{i=1}^{I} \frac{X_i^2(x_o)}{\omega_i} \sin\omega_i \bar{t}\right] \quad , (40)$$

with the eigenfunctions $X_i(x_o)$ from eq. (11) and ω_i from table II. Taking I=8 terms of the series, one obtains the maximum acceleration of the reactor as a function of the duration $\bar{t}$ of the first impact phase. This is plotted in Fig. 5 as a multiple of the acceleration due to gravity. As mentioned above, in all Ger-

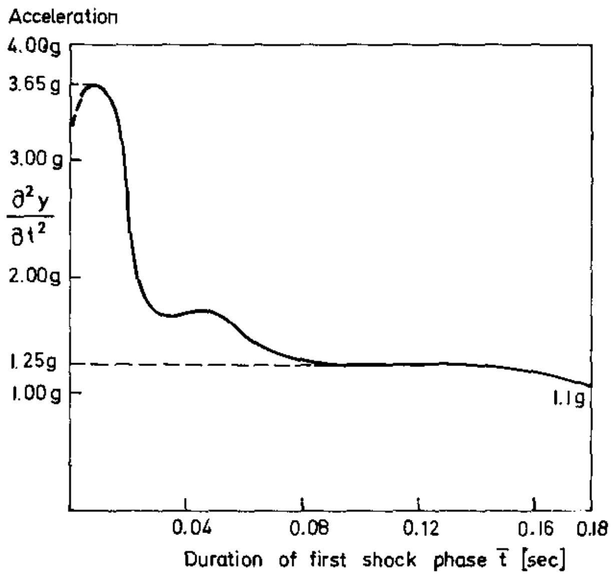

Fig.5 Maximum acceleration $\partial^2 y/\partial t^2$ at $x_o = 0.64L$ caused by an impact force P growing linearly within $\bar{t}$ sec to its maximum value $P_o = 400$ MN (g = acceleration due to gravity)

man collision tests with models of the struck ship according to table I the duration of the first collision phase was $\bar{t} > 0.1 \cdot \bar{T} = 0.088$ sec. Therefore, the maximum accelerations of up to 3.65 g in Fig. 5 need not be taken into serious consideration. It seems that, in this case, an acceleration of 1.25g is a good approximation.

8.3. Solution for the Entire Impact

Starting with the Fourier analysis of section 5.2., the impact force is expanded into a series with N=3 terms as in eq.(32). In the first shock phase, this force law can be very well approximated by a linear law for which the force rises within $\bar{t} = 0.18$ sec. to its maximum value of $P_o = 400$ MN (see section 8.1 and Fig.3b and 5). The entire duration of the impact is chosen from section 8.1 as
$\bar{T} = T + 2t^* = 0.676 + 2 \cdot 0.102 = 0.88$ sec.
Figs.6 and 7 show the corresponding displacements $w(x_o,t)$ and the relative accelerations $\partial^2 w/\partial t^2$ at the point x_o of the impacting force, obtained by numerical differentiation of eq.(36) with I=2. The time t=0 (zero-value of the Fourier-expansion) is of interest only for the calculation procedure. Impact begins at the time $t = -t^* = -0.102$ sec. and ends at $t = 0.78$ sec. In Figs.6 and 7 the rigid body displacements and accelerations are displayed by dotted lines so that the respective total values can be read off as the differences between the dotted curves and the curves of $w(x_o,t)$ and $\partial^2 w/\partial t^2$ for x_o,t. The total acceleration of 1.1 g at $\bar{t} = 0.18$ sec. of the first impact phase (t = 0.08 sec.) coincides with the corresponding value of Fig.5. During the impact, 1.35 g is the maximum total acceleration.

Whether the acceleration of nearly 2 g after the end of the impact will be reached seems to be questionable because damping will play a role with increasing time but this has not been considered in the foregoing investigation.

8.4. Stress Analysis

For calculating the bending moment, eq.(37) is used while the shear force is calculated from eq.(38) because the equivalent values from eqs.(34) and (35) in the first collision phase ($t < \bar{t}$), according to section 6.1., are negligibly small (see Figs.8 and 9). Further, only the dominating first two terms of B_{iN} are needed (i.e. set I=2 in the said equations) in order to obtain satisfactory approximations. The point $x_o = 0.64L$ of the impacting force remains as before, but for x we use the most endangered cross section (just in front of the reactor-room), namely $x = 0.59$ L. Then finally, the time dependencies of bending

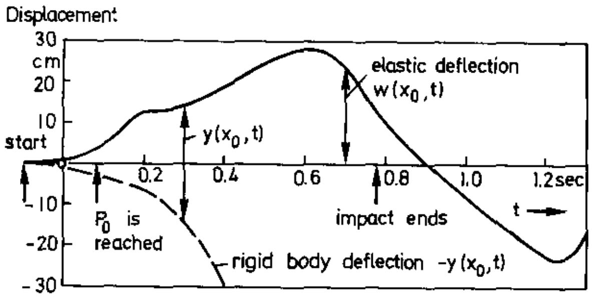

Fig.6 Displacement of the ship's axis at $x_o = 0.64L$ for a maximum shock force $P_o = 400$ MN reached within 0.18 sec. and acting at x_o

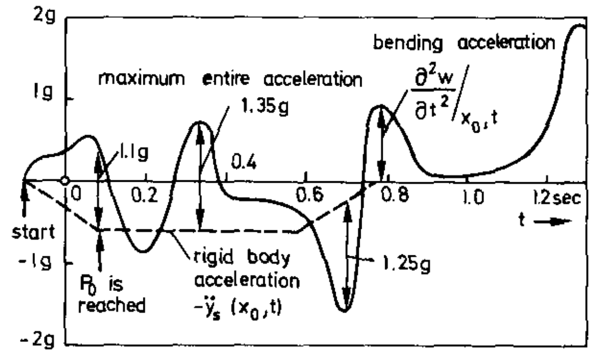

Fig.7 Accelerations at $x_o = 0.64L$ for maximum shock force $P_o = 400$ MN reached within 0.18 sec.(g=acceleration of gravity

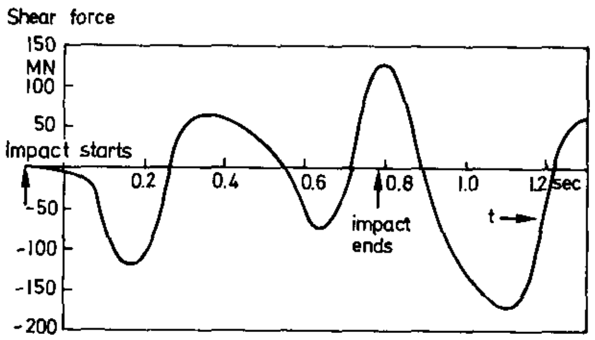

Fig.8 Shear force Q(t) at $x = 0.59L$ resulting from maximum shock force $P_o = 400$ MN acting at $x_o = 0.64L$

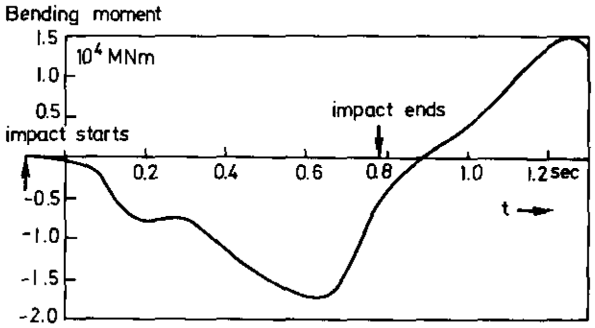

Fig.9 Bending moment $M_z(t)$ resulting from shock force as in Fig.8

moment and shear force follow from Figs. 8 and 9.

Only longitudinal stresses and bending shear stresses have been taken into consideration in the stress analysis. These have been combined into an equivalent uniaxial stress. Longitudinal stresses are caused by the impact bending moment M_z in the (x,y)-plane (see fig.9), by the still water bending moment in the (x,z)-plane, and by restrained warping associated with the torsion moment. The warping stresses and bending shear stresses have been calculated by assuming an open cross section and total warping restraint, which are admittedly rather strong simplifications. Fig.10a shows the double hull cross section just in front of the reactor room. The results of these calculations are plotted in Fig.10b as curves of longitudinal stresses versus time for the four most endangered points of the cross section.

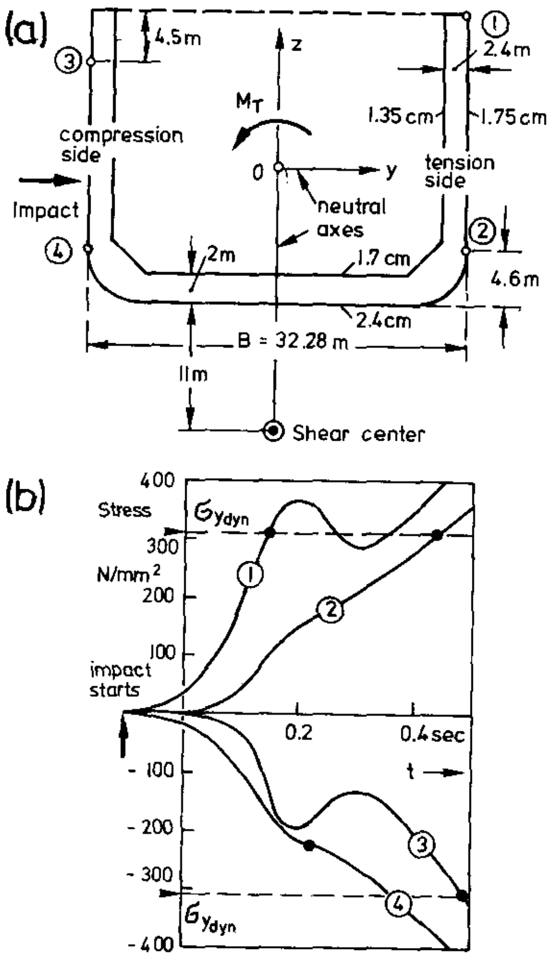

Fig.10 (a) Most endangered cross section at x = 0.59L
(b) Resultant uniaxial stresses in some of the most endangered points ① to ④ of the cross section (• = collapse)

Collapse is considered to start when the equivalent uniaxial stress reaches the dynamic yield stress $\sigma_{y_{dyn}}$ = 310 N/mm (45 ksi), or when buckling occurs on the compression side (e.g. point ④, taking the influence of the shear stresses on buckling into consideration). Collapse of the cross section will begin at about t = 0.2 sec., i.e. o.3 sec. after the impact started, if one considers only longitudinal and bending shear stresses. Inspection of Fig.9 shows that the bending moment $M_z(t)$ at t = 0.63 sec. theoretically is twice its value at the collapse time, so one can conclude that in reality a smaller impact force or a smaller collision velocity will cause collapse.

Inclusion of torsional shear stresses, whose calculation requires a more exact and rather complicated stress analysis of the whole ship which goes beyond the scope of this paper, will make the situation even worse.

Finally, the influence of the wave propagation of stresses on the impact process has been estimated. With most unfavorable assumptions, the maximum time for building up the final stress state in the whole ship is about 0.05 sec. Comparing this value with the stress-time curves of Fig.10, one may conclude that the time for propagation of the stress waves is so short that it has no essential influence on the impact process.

9. CONCLUSIONS

It must be emphasized that the preceding studies were not intended to predict exactly the behavior of the whole ship when struck in a collision because the assumptions for a collision scenario cannot be defined very accurately. The calculations give only a rough estimation. Therefore, the simplifications (constant bending rigidity, homogeneous mass distribution, linearly increasing shock force up to an approximately constant value and inclusion in the stress analysis of only longitudinal stresses and bending shear stresses) seem to be justified in order to get a rather simple and quick instrument for estimating the consequences of a collision impact. For the special case of a nuclear powered containership struck at the middle of its heavily protected reactor room, the study shows that in spite of - or perhaps even because of - the resistance of the protective construction, which is able to withstand the intruding of the striking ship's bow, the whole ship will collapse at a cross section in front of the reactor room. The study also shows that the maximum acceleration has so high a value that it must be taken into serious consideration in the design of the reactor facilities.

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