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Combining Extreme Environmental Loads for Reliability-Based Designs

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ABSTRACT AT
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INTRODUCTION

The nature of the present-day tools and procedures for calculating the response of a ship to environmental loads is such that only separate individual components of the response can be calculated successfully rather than the more relevant combined response. For example, several computer programs exist for calculating the stillwater bending moment and shearing forces using the usual static balance procedure, once the weight distribution and the ship geometry are specified. Other programs and tools exist for computing the vertical bending moment due to the motion of the ship in waves considering it as a rigid body. Although the same program may be used to calculate the horizontal and torsional moments, the results are usually given as separate entities and a special procedure must be adopted to relate them to each other to obtain the combined response. Similarly, secondary response due to hydrostatic pressure acting on portions of the hull are computed using other tools and procedures. Even within the higher frequency, primary dynamic loads and responses such as slamming and springing, computational

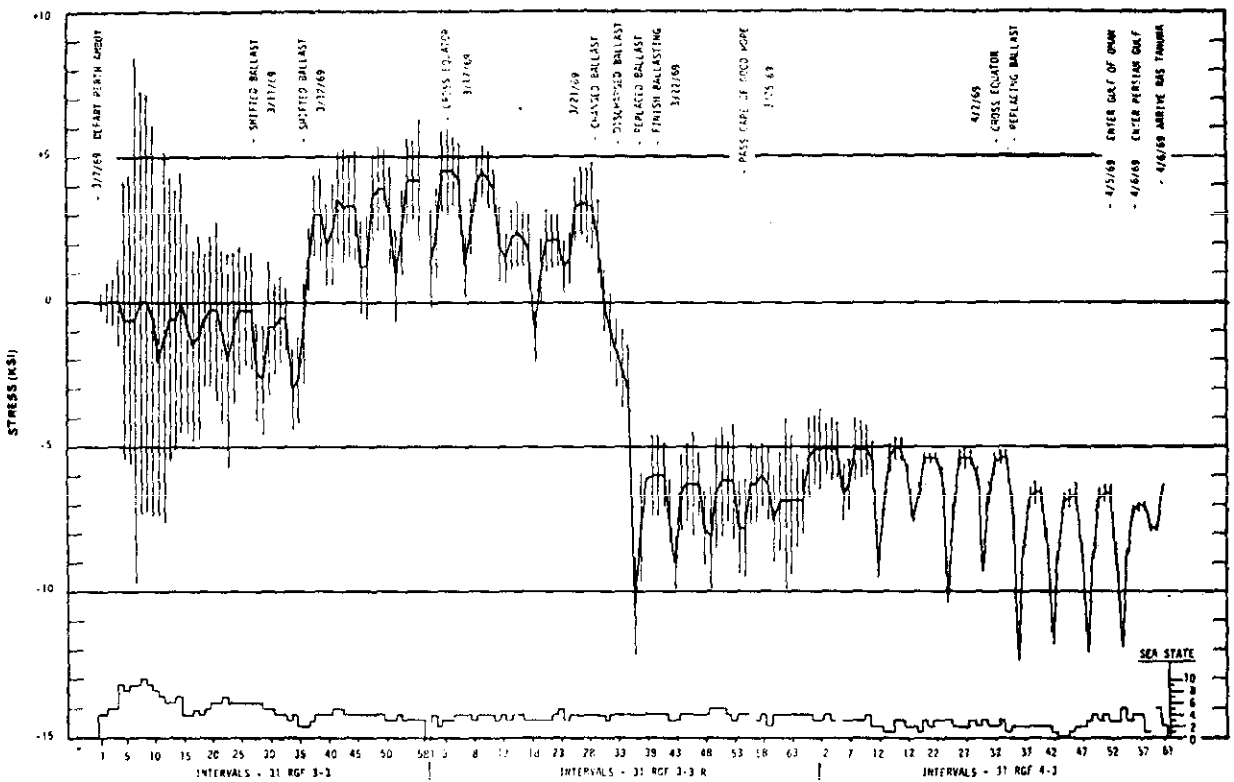

Fig. 1 Typical voyage variation of midship vertical bending stress for a bulk carrier. [1]

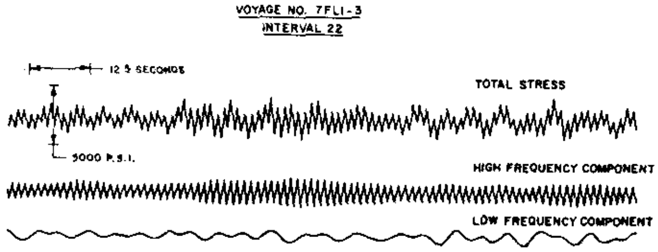

Fig. 2 Decomposition of a stress time history of a Great Lakes vessel into low and high frequency components.

methods are sufficiently different to give each a separate entity. The response due to thermal loads (e.g., air/water temperature variations) is another example where a drastically different tool such as a finite-element program may be necessary to obtain the response.

The reason for the separate calculations of the load/response components is not just due to computational difficulties. More importantly, the reason stems from differences in the basic analytical approaches to each load/response component and, whether or not, such approaches fall within certain analytical disciplines.

A ship designer is, therefore, faced with the important problem of computing the total response from the individual components, which are calculated using separate programs. For example, he may calculate the stillwater bending moment using the Ship Hull Characteristics Program, the wave vertical, horizontal and torsional moments from a rigid-body ship motion program, the higher frequency springing moment from a flexible-body vibration program, the slamming response from a slamming program, the thermal response using a finite-element program, etc. If all of these load/response components were static in nature, the problem would have been simple, because only the magnitude and direction of each load/response component would be necessary to obtain the total response. If the total response involves <u>dynamic cyclic</u> individual responses, then the problem becomes a little more complicated since attention must be given to the <u>phase</u> relation between the different response components. However, the actual problem that faces a ship designer, involves <u>dynamic random</u> individual responses that require the additional complications of determining the degree of <u>correlation</u> between the different components and the method of combining them which are the subjects of this paper.

Undoubtedly, there are certain similarities between decomposing response records of full-scale measurements into their basic components and combining analytically calculated components to obtain the total response. Since decomposing full scale measurements can be done with a certain degree of success, the approach taken in this paper is to invert the procedure in order to compute the combined response from the analytically determined components.

In the next section of this paper, a brief discussion is given of the decomposition of full-scale records into their basic components. In the following section, a method is developed to combine analytically-determined response components. In the following sections, application examples are given and some considerations for reliability-based designs are discussed together with some concluding remarks.

DECOMPOSITIONS OF MEASURED RECORDS INTO THEIR BASIC COMPONENTS

A typical measured stress time history of a bulk carrier is shown in figure 1 (from reference [1]). Usually, such a record consists of a rapidly varying time history of random amplitude and frequency, oscillating about a mean value. The mean value itself is a weakly time-dependent function and may shift from positive to negative (sagging to hogging). The two dominant factors which affect the mean value are (see references [8] and [9]):

1. The stillwater loads which can be accurately determined from the loading condition of the ship floating in stillwater.

2. The thermal loads which arise due to variations in ambient temperatures and differences in water and air temperatures.

A closer look at the rapidly varying part shows that it also can be decomposed into components. Figures 2 and 3 illustrate records taken over shorter periods of time (larger scale). Two main central frequencies appear in these records. The smaller central frequency is associated with the loads resulting from the motion of the ship as a rigid body (primarily heave and pitch motions). This lower central frequency is, therefore, close in magnitude to the wave encounter frequencies for wave length nearly equal

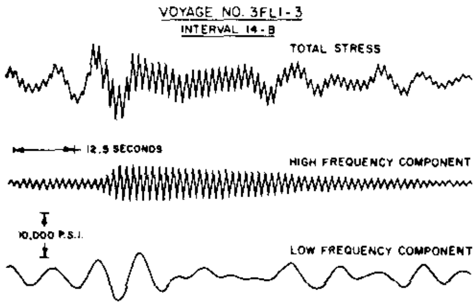

Fig. 3 Decomposition of a stress time history of an ocean going bulk carrier.

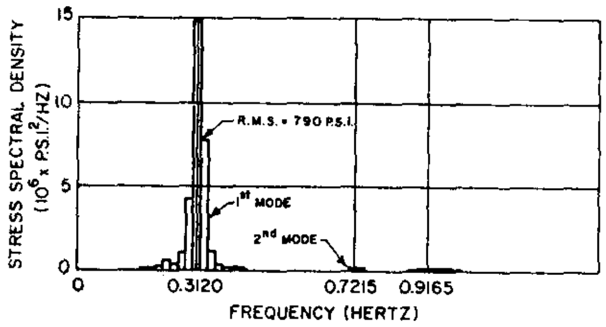

Fig. 4 Stress response spectrum of a large Great Lakes vessel. [2]

to ship length. The higher central frequency is associated with loads resulting from the two-node mode response of the ship when it flexes as a flexible body. This higher central frequency is thus close to the two-node mode natural frequency of the ship. The high frequency response itself can be due to "springing" of the flexible ship when excited by the energy present in the high-frequency wave components as shown in figure 2. It can be due, also to the impact of the ship bow on the water as the ship moves into the waves, i.e., slamming (possibly together with low-speed machinery-induced vibrations), see figure 3. Though springing and slamming may occur simultaneously, it is unusual to see records which exhibit both clearly. These two responses can be distinguished from each other by inspecting the records' envelope. In general, a decaying envelope (see figure 3) indicates a slamming response whereas a continuous envelope of varying amplitude, as shown in figure 2, indicates a springing response.

The rigid body and the high frequency responses do not always occur simultaneously in the same record. Quite often only the rigid body response appears in a record; particularly, in that of a smaller ship which has a high two-node mode frequency. Occasionally, only the high frequency springing response appears in a record when a ship is moving or resting in relatively calm water. In particular, long flexible ships with low natural frequencies as those operating in the Great Lakes do occasionally exhibit such records when operating in calm water or in a low sea state composed mainly of short waves. Under these conditions, a long ship will not respond as a rigid body to the short waves, but the two-node mode frequency of the hull can be sufficiently low to be excited by the energy content of these short waves. Figure 4 (from reference [2]) shows a measured response spectrum of a large Great Lakes vessel where the response is purely in the two-node mode[1] and higher frequencies with no rigid body response appearing in the spectrum. The figure shows that response at higher modes than the two-node mode can be measured, although small and relatively unimportant in most cases.

Slamming response on the other hand never occurs separately without rigid body response since, obviously, it is a result of the rigid motion of the ship in the waves.

COMBINING ANALYTICALLY DETERMINED RESPONSE COMPONENTS

An inverse procedure, of that for decomposing full scale measurements into the individual components, can be adopted for combining individually calculated response components. Two main steps should be used in the procedure for combining primary responses.

Step 1. To combine the low frequency wave-induced responses (rigid body) with the high-frequency responses (springing or slamming).

Step 2. To add the mean value to the response resulting from Step 1. The mean value consists of the stillwater and the thermal responses.

Step 1:

Consider an input-output system in which the input is common to several components of the system; it is required to determine the sum of the individual outputs. Here, the input represents the waves which can be in the form of a time history if a time-domain analysis is sought, or in the form of a sea spectrum if a frequency-domain evaluation is preferred.

[1] The two-node mode is labeled in the figure as the first mode.

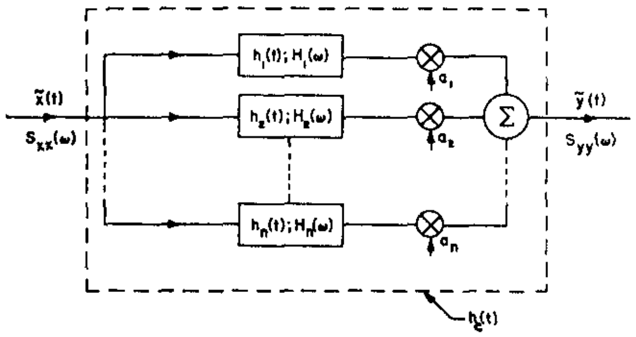

Fig. 5 Schematic representation of a multiple system with common input.

The components of the system represent the components of the load response of the ship to waves, e.g., the low-frequency wave-induced responses which consist of vertical, horizontal and torsional moments, the high frequency responses such as the springing loads, etc. It is required now to determine the sum of these component responses, i.e., to determine the output taking into consideration the proper relations or the appropriate correlation of the response components (see references [8] and [9]).

Schematically, the procedure is represented by figure 5. In this figure, "n" parallel linear components are considered which have common input[2] $\tilde{x}(t)$ and are summed up at the output to form $\tilde{y}(t)$. The output of each system is multiplied by a constant $a_i (i = 1, 2, \ldots n)$ before summing up all the components at a common node to form $\tilde{y}(t)$. These constants a_i give additional flexibility in the application of the model and can be used to "weigh" the contribution of each linear system to the sum.

In a time domain, the output $\tilde{y}(t)$ is given by the sum of the convolution integral of each system.

$$\tilde{y}(t) = \sum_{i=1}^{n} a_i [\int_0^{\infty} h_i(\tau)\tilde{x}(t-\tau)d\tau]$$

$$\int_0^{\infty} h_c(\tau)\tilde{x}(t-\tau)d\tau \qquad (1)$$

where

$$h_c(\tau) = \sum_{i=1}^{n} a_i h_i(\tau) \qquad (2)$$

$h_i(\tau)$ is the impulse response function of each linear system, i.e., the response of each linear system to unit excitation multiplied by time (response to the Dirac delta function). $h_c(\tau)$ is a composite impulse response function which sums the responses of the individual components.

[2] A tilda over a variable indicates a random variable.

It should be noted that the impulse response functions of the individual components "$h_i(\tau)$" may or may not be easy to obtain depending on the complexity of the system. With suitable instrumentation, it is sometimes possible to obtain a good approximation to $h_i(\tau)$ experimentally. For the ship system, $h_i(\tau)$ can be determined for most load components.

In a frequency domain analysis, a similar procedure can be used. In fact, since the system function $H_i(\omega)$ is simply the Fourier transform of $h_i(t)$, i.e.,

$$H_i(\omega) = \int_0^{\infty} h_i(t)e^{-j\omega t}dt \qquad (3)$$

therefore, we can define a composite system function $H_c(\omega)$ as

$$H_c(\omega) = \int_0^{\infty} h_c(t)e^{-j\omega t}\,dt$$

$$= \sum_{i=1}^{n} a_i H_i(\omega) \qquad (4)$$

It should be noted that for a single system, the relation between the input spectrum and the output spectrum is given by the usual relation:

$$[S_{yy}(\omega)]_i = S_{xx}(\omega)H^*_i(\omega)H_i(\omega)$$

$$= S_{xx}(\omega)|H_i(\omega)|^2 \qquad (5)$$

where $S_{xx}(\omega)$ is the sea spectrum which represents a common input, $[S_{yy}(\omega)]_i$ is a response spectrum of an individual load component, $H^*_i(\omega)$ is the complex conjugate of the system function of a response component.

The modulus of the individual system function $|H_i(\omega)|$ is the response amplitude operator of the individual response components, i.e.,

$$[\text{R.A.O.}]_i = [\rho(\omega)]_i = |H_i(\omega)| \qquad (6)$$

and, therefore, equation (5) represents the familiar relation between the input and the output spectra of a single linear system.

For our composite system, an equation similar to equation (5) can be determined for the n-response components and the "weight" factors "a_i" as follows

$$S_{yy}(\omega) = S_{xx}(\omega)H^*_c(\omega)H_c(\omega)$$

$$= S_{xx}(\omega)\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j H_i(\omega)H^*_j(\omega)$$

$$(7)$$

The double sum in equation (7) can be expanded such that a final expression for the total response spectrum $S_{yy}(\omega)$, which combines the individual response spectra, may be written in the form:

$$S_{yy}(\omega) = [\sum_{i=1}^{n} a_i^2 |H_i(\omega)|^2] S_{xx}(\omega)$$

$$+ [\sum_{i=1}^{n} \sum_{\substack{j=1 \\ (i \neq j)}}^{n} a_i a_j H_i(\omega) H^*_j(\omega)] S_{xx}(\omega) \qquad (8)$$

It should be noted that the first term in equation (8) represents simply the algebraic sum of the individual response spectra, each modified by the factor a_i. The second term, which can be either positive or negative, represents a corrective term which depends on the correlation between the load components as can be seen from the multiplication of $H_i(\omega)$ by the complex conjugate of $H_j(\omega)$.

If the system functions $H_i(\omega)$ do not overlap on a frequency axis (i.e., disjoint systems), that is, if

$$H_i(\omega) H^*_j(\omega) \simeq 0 \qquad (9)$$

then the second term in equation (8) becomes zero and the load components are uncorrelated. In this case, the total spectrum is simply the algebraic sum of the individual spectra of the load components, modified by the factors a_i. Furthermore, if the wave input is considered to be a normal random process with zero mean, as usually is the case, then the respective output load responses are jointly normal and are independent. Thus, the total response, in this special case, is a zero mean normal process with a mean square value given by:

$$\sigma_y^2 = \int_0^\infty S_{yy}(\omega) d\omega$$

$$= \sum_{i=1} a_i^2 \int_0^\infty |H_i(\omega)|^2 S_{xx}(\omega) d\omega \qquad (10)$$

In the more general (and more realistic) case where some or all of the response components are correlated, the mean square is given by

$$\sigma_y^2 = \int_0^\infty S_{yy}(\omega) d\omega$$

$$= \sum_{i=1}^{n} a_i^2 \int_0^\infty |H_i(\omega)|^2 S_{xx}(\omega) d\omega$$

$$+ \sum_{i=1}^{n} \sum_{\substack{j=1 \\ (i \neq j)}}^{n} a_i a_j \int_0^\infty H_i(\omega) H^*_j(\omega) S_{xx}(\omega) d\omega \qquad (11)$$

It should be noted that in equations (10) and (11) the mean square is taken equal to the variance of the combined response since the mean value of the wave responses is usually very small. As noted earlier, the other responses which consist mainly of the stillwater, and the thermal loads will be added later in the second step of the analysis to form a mean value for step 1 combined responses.

In connection with equations (10) and (11) it should be mentioned that the usual Rayleigh multiplier used to estimate certain average quantities, such as average of the highest one third, one tenth response, etc., are not generally applicable in the case of the combined response, since these multipliers are associated with a narrow-band spectrum for which the amplitudes can be represented by a Rayleigh distribution. It is only in the case where each of the load component responses is narrow-band and each happen to be closely concentrated around a common central frequency "ω_o", that it would be reasonable to conclude the combined response $S_{yy}(\omega)$ is, itself, a narrow-band process.

In the more general case, where the combined response spectrum is not a narrow-band spectrum, the various statistical quantities can be determined from a more general distribution which includes the Rayleigh distribution as a special case. The general distribution can be found in [3,4], and the factors which, when multiplied by the root mean square, give the various statistical quantities, can be found in [4,5].

Equation (11) can be written in a different form which is more convenient to use in applications, and which makes it easier to define the correlation coefficients between the different response components.

$$\sigma_y^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ (i \neq j)}}^{n} a_i a_j \rho_{ij} \sigma_i \sigma_j \qquad (12)$$

where

$\sigma_i^2 = \int_0^\infty |H_i(\omega)|^2 S_{xx}(\omega) d\omega$ = variances or mean squares of the response spectra of the individual load components. (13)

ρ_{ij} = correlation coefficients of individual load components defined by,

$$\rho_{ij} = \frac{1}{\sigma_i \sigma_j} \int_0^\infty H_i(\omega) H^*_j(\omega) S_{xx}(\omega) d\omega \qquad (14)$$

Equation (12) with definitions (13) and (14), which are derived from equation (11), form the basis for combining the step 1 responses of a ship hull girder in a frequency domain analysis taking into consideration the correlation between the response components. If the response components are uncorrelated, i.e., if $\rho_{ij} = 0$, the second term in equation (12) drops out and the variance of the combined responses (output) is simply the algebraic sum of the individual variances modified by the factors a_i. As discussed earlier, this occurs when the system function of the various components do not overlap in frequency or overlap in a frequency range where the individual responses are small. On the other hand, if the individual components are perfectly correlated, ρ_{ij} may approach plus or minus unity and the effect of the second term of equation (12) on the combined load variance σ_y^2 can be substantial.

The physical significance of the correlation coefficient can be further illustrated by considering only two response components for simplicity. If ρ_{12} is large and positive (i.e., approaching +1), the values of the two response components tend to be both large or both small at the same time, whereas if ρ_{12} is large and negative (i.e., approaching -1), the value of one response component tends to be large when the other is small, and vice versa. If ρ_{12} is small or zero, there is little or no relationship between the two response components. Intermediate values of ρ_{12} between 0 and ±1 depend on how strongly the two responses are related. For example, the correlation coefficient ρ_{12} of the vertical and horizontal bending moments acting on a ship is expected to be higher than of the vertical and springing moments since the overlap of the system functions in the latter case is smaller than in the former case.

In a time domain analysis, the convolution integral represented by equation (1) with the composite impulse response function as given by equation (2) form the basis for determining the combined load response. The question of whether the time or the frequency domain analysis should be used depends primarily on what form the required input data is available. In general, most wave data and practical analysis are done in a frequency domain, although in some cases where slamming loads are a dominant factor, it may be advisable to perform the analysis in time domain.

Step 2:

In this step, the stillwater and the thermal responses should be combined to form the mean value for the rigid body motion and higher frequency responses. The stillwater and thermal responses are weakly time-dependent variables so that in a given design extreme load condition they can be considered constants, say over the duration of a design storm. Therefore, these two responses can be treated as static cases and can be combined for one or several postulated design conditions without difficulty. Alternately, if statistical data are available for each of these responses, the mean and variance of the combined response can be easily determined.

The stillwater response can be accurately determined for all loading conditions using computer programs such as the Ship Hull Characteristics Program. Several postulated, extreme but realistic, weight distributions can be assumed in the final stages of design, and the corresponding stillwater response can be computed.

If a statistical description of the stillwater bending moment is adopted, data have shown that the general trend assumes a normal distribution for conventional types of ships. A sample histogram based on actual ship operation data for a containership from reference [6] is shown in figure 6. A mean value of the stillwater bending moment can be estimated based on this histogram for all voyages, or for a specific route such as inbound or outbound voyages.

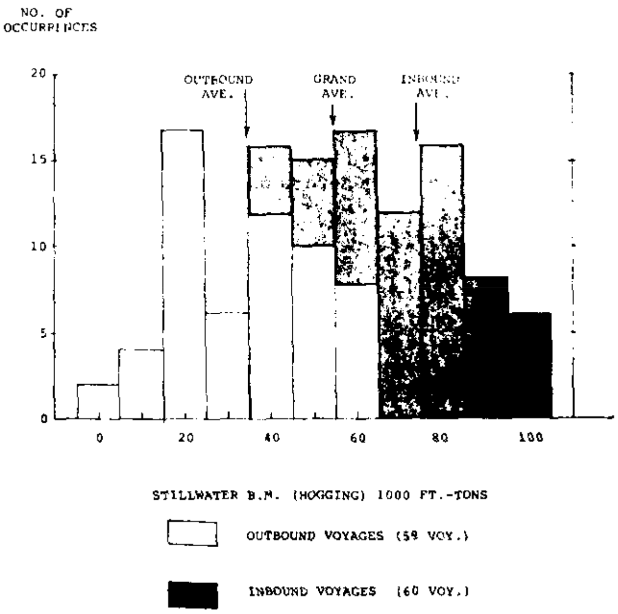

Fig. 6 Histogram of stillwater bending moment of a containership.

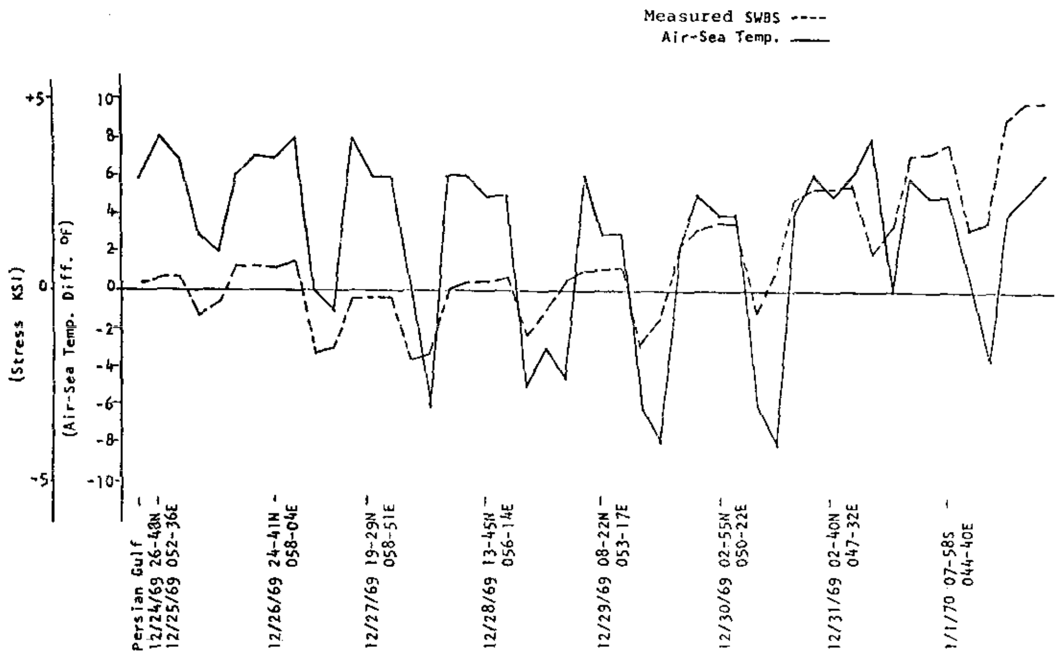

Fig. 7 Correlation between measured stillwater bending moment and air/sea temperature difference.

These mean values together with a set of standard deviations, which can also be estimated from the histogram, can be utilized to determine the extreme total moment using a statistical approach.

Since the stillwater response, the rigid body motion response, and the higher frequency responses are all functions of the ship weight and its distribution, it is preferred that the combined response be calculated for a group of selected loading conditions (and selected temperature profiles).

Primary thermal response is usually induced by differences in water/air temperatures and by variations in ambient temperatures. Figure 1 shows that a consistent diurnal stress variation of magnitude of 3-5 ksi has been recorded onboard a bulk carrier. A study of full-scale stress data measured on a larger tanker indicates that the diurnal stress variations correlate well, as shown in figure 7 with temperature differentials beteen air and sea.

Taking the North Atlantic route as an example, the average diurnal change of air temperature is about 10° F. The total diurnal change of deck plating temperatures may vary from 10° to 50°F, depending upon the cloud cover conditions and the color of the deck plating. For estimating the thermal loads on a ship hull, the sea temperature may be assumed as constant. Once the temperature differential along a ship hull is determined, the thermal stresses can be calculated, using either a general purpose finite element computer program or a simplified two-dimensional approach. The maximum thermal response may be then added to the stillwater response for certain postulated design conditions to form the mean value for the low and high frequency dynamic responses.

Although high thermal responses may not happen in high seas, a heavy swell may possibly occur under a clear sky. Therefore, several temperature conditions are to be taken into consideration in determining the combined design response.

APPLICATION EXAMPLES

Since step 2 of combining the environmental loads acting on a ship imply a simple superposition of static loads (either as constants or as random variables) only step 1 of combining the dynamic random loads is considered in the following application examples.

Generally, large tankers travelling in oblique seas may encounter horizontal bending moments of the same order of magnitude of the vertical bending moments. Therefore, the combined effect of the vertical and horizontal moments can be critical under certain conditions. The distribution of the primary stress in the deck as a result of the combined effect becomes non-uniform and assumes a maximum value at one edge. The combined stress at the deck edge, σ_c is given by

$$\tilde{\sigma}_c = \frac{\tilde{M}_v(t)}{S_v} + \frac{\tilde{M}_h(t)}{S_h} \qquad (15)$$

where,

$\tilde{\sigma}_c$ = combined edge stress

$\tilde{M}_v(t)$ = the vertical bending moment component

$\tilde{M}_h(t)$ = the horizontal bending moment component

S_v, S_h = the vertical or the horizontal section modulus, respectively.

Defining the combined moment as the combined edge stress multiplied by the vertical section modulus and using equation (15), we can write,

$$\tilde{M}_c(t) = \tilde{\sigma}_c(t) S_v = \tilde{M}_v(t) + k\tilde{M}_h(t)$$

where,

$$K = \frac{S_v}{S_h} \quad . \qquad (16)$$

Applying equations (12), (13) and (14) and extending the results for the case of a two-dimensional sea spectrum, the mean square value of the combined response $\sigma_{Mc}{}^2$ can be written as (see equation (12)):

$$\sigma_{Mc}{}^2 = \sigma_{Mv}{}^2 + K^2\sigma_{Mh}^2 + 2\rho_{vh}K\sigma_{Mv}\sigma_{Mh} \qquad (17)$$

where $\sigma_{Mv}{}^2$ and $\sigma_{Mh}{}^2$ are the mean square values of the vertical and horizontal bending moments, respectively, given by equation (13) as[3]:

$$\sigma_{Mv}{}^2 = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{\infty} S_{xx}(\omega,\mu)\,|H_v(\omega)|^2 \, d\omega d\mu \qquad (18)$$

$$\sigma_{Mh}{}^2 = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{\infty} S_{xx}(\omega,\mu)\,|H_h(\omega)|^2 \, d\omega d\mu \qquad (19)$$

Using equation (14) the correlation coefficient is defined as:

$$\rho_{vh} = \frac{1}{\sigma_{Mv}\sigma_{Mh}} \cdot \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{\infty} S_{xx}(\omega,\mu) H_v(\omega) H^*_h(\omega)\, d\omega d\mu \qquad (20)$$

[3] For simplicity of notation, the dependence of the R.A.O.'s $|H_v(\omega)|$ and $|H_h(\omega)|$ on the ship headinging with respect to wave components is dropped from the notation.

The Response Amplitude Operators $|H_v(\omega)|$ and $|H_h(\omega)|$ and more generally, the system functions $H_v(\omega)$ and $H_h(\omega)$ can be determined from any typical rigid-body ship motion computer program.

It should be noted that in equation (12), the coefficient a_1 was taken as unity and a_2 was taken equal to

$$a_2 = K = \frac{S_v}{S_h}$$

(see equation (16)) in determining equation (17). It should be also noted that the integrand in equation (20) contains the information regarding the phase between the horizontal and vertical moments and that the integration of such information with respect to frequency and the angle μ between a wave component and the prevailing wave system leads to the determination of the correlation coefficient as given by equation (20). Finally, the root-mean-square (r.m.s.)$_c$ of the combined edge stress is given by

$$(\text{rms})_c = \frac{\sigma_{Mc}}{S_v} \qquad (21)$$

where σ_{Mc} is the r.m.s. of the combined moment given by equation (17).

The r.m.s. of the combined moment and the correlation coefficient as given by equations (17) and (20), respectively, were computed in reference [5] for a large tanker of DWT 327,000 tons. The r.m.s. values of vertical and horizontal moments were computed using a rigid body ship motion computer program and combined using equation (17) to obtain the r.m.s. of the combined moment. The results of the calculations are plotted in figures 8 and 9 versus the ship heading for the cases of long-crested and short-crested waves, respectively. Significant wave heights of 7.4, 19.2 and 43.3 feet were considered in order to examine the general behavior of the responses in low, moderate and high sea states. The discussion of these results in reference [5] indicated that the horizontal bending moment is not small compared with the vertical bending moment (see figure 9). In severe seas, the maximum response of the combined moment (and the vertical moment) is in head and following seas. Figure 10 shows the variation of the combined bending moment with the sea state as obtained by three different methods. The first is based on equation (17) with σ_{Mv}, σ_{Mh} and ρ_{vh} given by equations (18), (19), and (20) respectively. The second method is based on equations (17), (18), and (19) also, but with a correlation coefficient ρ_{vh} equal to 0.32 obtained from the 1973 ISSC Proceedings (determined emperically). The third method is based on ρ_{vh} = 0.53

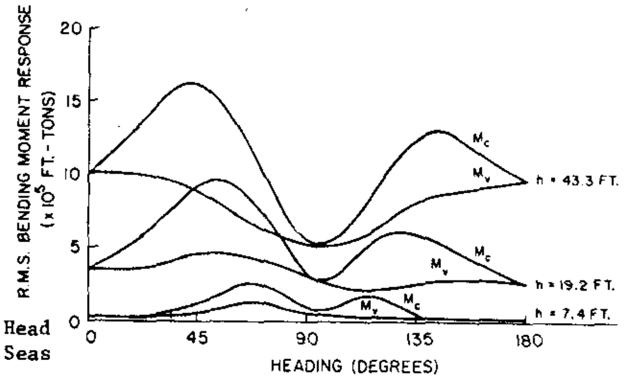

Fig. 8 Variation of vertical and combined wave bending moments with heading in long-crested seas.

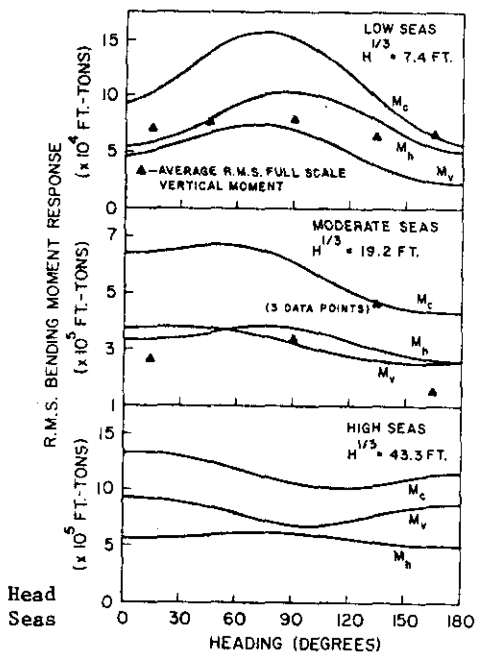

Fig. 9 Variation of vertical, horizontal and combined moments with heading in short-crested seas.

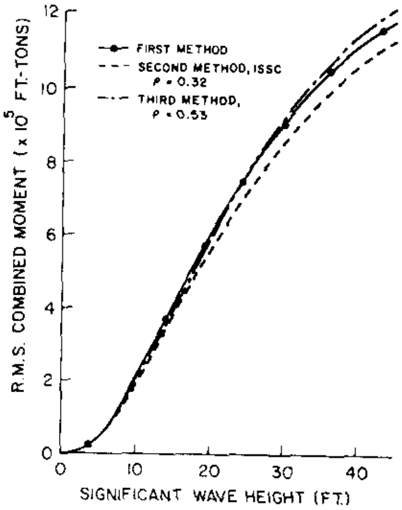

Fig. 10 Comparison of combined bending moment responses as computed by three methods.

as determined by averaging the responses in short crested seas as determined from equation (17) for all headings and for the three representative sea states. The mean value of ρ_{vh} obtained in this manner was 0.53, significantly higher than the ISSC value. However, the effect of ρ_{vh} on the r.m.s. combined moment is small, as can be seen from figure 10.

As a second application example, the combined vertical and springing moment will be considered next. Using frequency domain analysis and using equations (12), (13) and (14) with $a_1 = a_2 = 1$, we obtain the mean square value of the combined response "$\sigma_{vs}{}^2$" in long-crested seas as

$$\sigma_{vs}{}^2 = \sigma_v{}^2 + \sigma_s{}^2 + 2\rho_{vs}\sigma_v\sigma_s \qquad (22)$$

where,

$\sigma_v{}^2$ = mean square of vertical bending moment

$$= \int_0^\infty S_{xx}(\omega)\,|H_v(\omega)|^2 d\omega \qquad (23)$$

$\sigma_s{}^2$ = mean square of the springing moment

$$= \int_0^\infty S_{xx}(\omega)\,|H_s(\omega)|^2 d\omega \qquad (24)$$

ρ_{vs} = correlation coefficient

$$= \frac{1}{\sigma_v\sigma_s}\int_0^\infty S_{xx}(\omega)H_v(\omega)H^*_s(\omega)\,d\omega \qquad (25)$$

and $H_v(\omega)$ and $H_s(\omega)$ are the complex system functions of the vertical wave moment and springing moment respectively. The system response functions can be computed using computer programs which take into consideration the effects of ship flexibility in the response, such as the springing-seakeeping program "SPRINGSEA", [5]. Application of equations (22), (23) and (24) to several Great Lakes Vessels where springing is important is given in references [5] and [7] and will not be further discussed here. These equations together with equations (1) and (2) for the time-domain analysis have a wide range of applicability to any two or more dynamic random responses including, combining primary and secondary[4]

[4] Such as primary inplane loads on grillages due to overall bending of the hull and secondary lateral pressure arising from the randomly varying water surface.

responses [5], vertical, and torsional moments for ships where torsion is important, high frequency loads with vertical and horizontal moments, etc. In all of the cases, the coefficients a_i must be appropriately determined. When determining the various statistical averages from the combined r.m.s. values, the more general distribution given in [3], [4], and [5] for the peaks should be used instead of the usual Rayleigh distribution.

DESIGN CONSIDERATIONS

For design purposes, operational envelopes must be postulated to project the loads and responses of the hull. The operational envelopes normally include design variables such as sea state, ship speed and heading, ship loading condition, and thermal condition. A good discussion of this is given in references [8] and [9]. For a reliability-based design, either a short- or long-term procedure may be used. In the short-term procedure, only one or two severe design storms (sea states) are considered together with one or two ship loading condition and thermal condition. In the long-term procedure, the entire mission profile of the ship must be postulated including ship route, sea states expected to be encountered during operation, expected total years of service, etc. In either procedure, if a strict probabilistic approach is used, then the extreme values of the combined response must be estimated using a probability measure and compared with the ultimate strength of the hull to determine the reliability level or the risk of failure. The basic methodology of these two procedures was developed in reference [10] and is described in more detail in that reference for loads that consider the static stillwater and the dynamic (rigid body) vertical wave bending moments only. The basic methodology can be extended to include the effects of the combined loads as described briefly in this section.

In recent applications of the probabilistic approach and structural reliability to ships, it has been found that the short-term procedure is useful because of its simplicity and because it is less demanding in terms of the required input data and analysis time. In fact, in a postulated design sea storm, ship loading condition and thermal condition, the expected maximum combined dynamic bending moment "BM_{max}" in N maxima[5] during the storm, can be calculated from [2]:

$$E[BM_{max}] = \sqrt{2m_o}\{[\ln(1-E^2)^{1/2} N]^{1/2} + 0.2886[\ln(1-E^2)^{1/2} N]^{-1/2}\} \quad (26)$$

Where m_o is the zero moment or the area under the combined response spectrum of the dynamic loads, i.e., $m_o = \sigma_Y^2$ where σ_Y^2 is given by equations (11) or (12); E is the bandwidth parameter defined by

$$E^2 = 1 - \frac{m_2^2}{m_o m_4}$$

and

$$m_m = \int_0^\infty \omega^m S_{YY}(\omega)d\omega$$
$$= \sum_{i=1}^{n} a_i^2 \int_0^\infty \omega^m |H_i(\omega)|^2 S_{XX}(\omega)d\omega$$
$$+ \sum_{i=1}^{n} \sum_{\substack{j=1 \\ (i\neq j)}}^{n} a_i a_j \int_0^\infty \omega^m H_i(\omega) H^*_j(\omega) S_{XX}(\omega)d\omega \quad (27)$$

The static stillwater and thermal responses should be then added to the expected maximum combined dynamic response given by equation (26) before including it in any safety analysis (see for example reference [6]). Other methods within the short-term analysis can be used such as the distribution-free method given in reference [11] in conjunction with equations (8), (11) and (12).

On the other hand, the long-term analysis is also useful and is essential if the fatigue fracture modes are to be included in the reliability analysis since the fatigue reliability part requires the full loading and response histories. If a long-term full probabilistic method is used, a general Weibull distribution may be suitable to represent the combined loads. The Weibull distribution is given by

$$f_X(x) = \frac{L}{k}\left(\frac{x}{k}\right)^{L-1} e^{-(x/k)^L} \qquad x \geq 0$$

$$F_X(x) = 1 - e^{-(x/k)^L} \qquad x \geq 0 \quad (28)$$

where

X = random variable representing the combined wave bending moment amplitude,

$f_X(x)$ = probability density function of X,

[5] The negative maxima should be included in N for use in equation (26).

$F_X(x)$ = distribution function of X,

L and k = Weibull distribution parameters.

The Weibull probability paper can be used to fit the data and determine the parameters L and k. For the commonly used Weibull paper, the slope of the straight line which fits the data is "L" and the intercept with the axis is "-L log k". Thus k and L can be determined. Other methods for estimating L and k can be used such as the method of moments or the maximum likelihood method.

Extrapolation of the available data to obtain the extreme values over the entire ship life can be estimated using several methods. One of these methods is based on order statistics. Using the Weibull distribution as an initial distribution and including the stillwater and thermal bending moments in the manner described earlier in this paper, the probability law of the total extreme bending moment Z_n can be written in the form:

$$\phi_{Zn}(z) = \frac{nL}{k}\left(\frac{z - m_1 - m_m}{k}\right)^{L-1} e^{-\left(\frac{z - m_1 - m_m}{k}\right)} \cdot \left[1 - e^{-\left(\frac{z - m_1 - m_m}{k}\right)^L}\right]^{n-1}$$

$$\Phi_{Zn}(z) = \left[1 - e^{-\left(\frac{z - m_1 - m_m}{k}\right)^L}\right]^n \qquad (29)$$

where

Z_n = random variable representing the total extreme bending moment,

m_1 = value of the stillwater bending moment for a selected loading condition,

m_m = value of the thermal bending moment for a selected temperature condition,

ϕ_{Zn} and Φ_{Zn} = the probability density and distribution functions of the random variable Z_n.

Equations (28) or (29) must be checked to see how well they fit the available data before they can be used in design. In fact, at the present time, it is recommended here to use a semi-probabilistic approach such as strength reduction and load magnification factors [6] or a distribution-free method [11] until the validity of extrapolation procedures and the fit of data in the distributions are demonstrated when sufficient data are collected.

CONCLUDING REMARKS

For reliability-based designs, the load element must include the stillwater loads, the thermal loads, the low-frequency wave induced loads, and the high-frequency loads. These loads can be combined as described in this paper, and the "maximum" combined load should reflect an envelope of all possible extreme values during the life of the vessel. The short- and long-term procedures of the reliability analysis are briefly described in the previous section for the case of the combined load as developed in this paper.

The second element in a reliability-based design is the strength or the ultimate carrying capacity of the hull. This has not been discussed in this paper, but reference [6] evaluates briefly the variables which introduce uncertainities in the hull strength.

At the present time, it is important that several levels be investigated in the structural reliability analysis of ships, whether the long-term or short-term procedure is used. The lack of long records and sufficiently large amounts of data necessitate such diversified analysis.

Three distinct levels of structural reliability may be used at the present time [6]. In the first level, which is closest to the procedures currently used in practice, the concept of a load magnification and a strength reduction factor is used. Such factors are determined from the standard deviation or the coefficient of variation of the available data which provides a measure of the uncertainty or variability in the design parameter. The higher the degree of uncertainty, the higher these factors are, and therefore, they are usually taken as multiples of the standard deviations of the variables. Reference [6] gives a more detailed discussion of this method.

The second level of structural reliability (the distribution-free method) also relies on computing only the mean and standard deviation of the total combined bending moment and of the ultimate collapse moment. The uncertainty in estimating the design parameters is thus reflected by the standard deviations of the variables. On this basis, a safety index can be determined (see reference [11]). The safety index can be mathematically related to the probability of failure if the probability distributions of the combined bending moment and the ultimate collapse moment are known. However, in the absence of such information, consistent designs can be achieved using the safety index on the basis of some, though unspecified, reliability level.

The third level of structural reliability is based on a strictly probabilistic concept. The combined extreme bending moment and the ultimate collapse moment are considered as random variables and are fitted into appropriate probability distributions such as given by equations (29) for the load. By this means, the reliability function or conversely the risk of failure can be constructed and used as a measure of safety.

Combining the static and dynamic loads as described in this paper or by any other means is an essential step in any of the three reliability levels briefly described above. It is recommended, however, that only the first two levels of the reliability analysis be used at the present time. The use of the strict probabilistic reliability concept is difficult at the present time because the available data are too limited to provide the exact forms of the probability distributions of the loads and strength or to provide adequate data to check the extrapolation procedures to the extreme values. The sample size required is of the order of multimillion pieces of records or data. In addition, the first two levels are consistent with the simplified hydrodynamic and structural analyses currently used to predict the sea loads and the structural response. Finally, the rules and procedures for designers must remain uncomplicated, thus simplified analysis such as given by the first two reliability levels is more appropriate.

REFERENCES

1. R. S. Little, E. V. Lewis and F. C. Bailey, "A Statistical Study of Wave Induced Bending Moments on Large Oceangoing Tankers and Bulk Carriers," Trans. SNAME, 1971.

2. M. Critchfield, "Evaluation of Hull Vibratory (Springing) Response of Great Lakes Ore Carrier M/V Stewart J. Cort," DTMB Report 4225, Nov., 1973.

3. S. O. Rice, "Mathematical Analysis of Random Noise," Bell System Technical Journal, Vol. 23, 1944 and Vol. 24, 1945.

4. D. Cartwright and M. Longuet-Higgins, "The Statistical Distribution of the Maxima of a Random Function," Proceedings of the Royal Society of London, Vol. 237, 1956.

5. S. G. Stiansen and A. E. Mansour, "Ship Primary Strength Based on Statistical Data Analysis," Trans. SNAME, 1975.

6. S. G. Stiansen, A. E. Mansour, H. Y. Jan and A. Thayamballi, "Reliability Methods in Ship Structures," The Naval Architect, Journal of RINA, July 1980.

7. S. G. Stiansen, A. E. Mansour and Y. N. Chen, "Dynamic Response of Large Great Lakes Carriers to Wave-Excited Loads," Trans. SNAME, 1978.

8. E. Lewis, D. Hoffman, W. Maclean, R. VanHooff and R. Zubaly, "Load Criteria for Ship Structural Design," Ship Structure Committee Report no. SSC-240, 1977.

9. J. Dalzell, N. Maniar and M. Hsu, "Load Criteria Application," Ship Structure Committee Report no. SSC-287, 1979.

10. A. E. Mansour, "Probabilistic Design Concepts in Ship Structural Safety and Reliability," Trans. SNAME, 1972.

11. A. E. Mansour, "Approximate Probabilistic Method of Calculating Ship Longitudinal Strength," Journal of Ship Research, Vol. 18, No. 3, Sept. 1974.

ABSTRACT

The application of probabilistic and reliability methods to ship hull structures requires the estimation of one or more extreme loading conditions. Each consists of components such as stillwater load, thermal load, low-frequency wave-induced load, and high-frequency springing or slamming load. Combining such a group of time-dependent load components of various frequencies has always presented a problem to ship designers. The stillwater and thermal loads, though (weakly) time-dependent variables, are of sufficiently different character from wave loads to necessitate separate treatment. The wave loads themselves consist of low-frequency wave-induced and high-frequency slamming or springing loads which require the summation of random time series of different frequency compositions. This paper examines and presents an approach suitable for determining the combined extreme response to the load components and the statistics of the sum.