PROBABILISTIC VULNERABILITY ASSESSMENT TOOL FOR SURFACE SHIP UNDER EXTREME DYNAMIC LOADS

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While probabilistic methods have been applied extensively to quantify uncertainty of sea environments and estimate reliability of naval vessels under normal seaway loads, their extension to surface ships subjected to extreme dynamic loads due to collision, grounding, and weapon effects has not been well explored. Under these extreme loads, the structural response behaves nonlinearly. In addition, the complexity of fluid-structure interaction phenomena may render the assumptions on the loading process (stationary and Gaussian) invalid. In this study, we develop probabilistic analysis tools to assess the response uncertainty and variance in vulnerability assessment due to known variability in materials properties, geometric configuration, failure criteria, and loading parameters.

INTRODUCTION

Current structural design of U.S. Navy ships is based on deterministic analysis methodologies and design rules/requirements, which are highly correlated with test data and at-sea experience. Experience exists both for seaway loads as well as for impulsive loads due to applied collision and wave slamming. To provide safe and strong ships for our fleet, the design rules/requirements are conservative. The extent of conservatism is, currently, not quantified.

Ship vulnerability assessment plays an important role in identifying the levels of operational capability that must remain after a ship is damaged under extreme dynamic loads. Naval ships must be survivable under a hostile environment and the ship survivability is determined by the mathematical complement of killability. The killability is defined as the product of the susceptibility and the vulnerability. Mathematically, the susceptibility is defined as the probability of being hit (P_H), while the vulnerability is given by the probability of being killed if hit ($P_{K/H}$). The reduction of P_H can be achieved with excellent defensive weapons, sensors, countermeasures, and reduced signatures; while the reduction of $P_{K/H}$ can be obtained with the aid of structural enhancement, advanced materials, and increased ability to absorb damage. As indicated in [1], at present, ship vulnerability reduction has not been given the same priority as susceptibility reduction, leading to unbalanced operational requirements. In addition, the present vulnerability assessment of surface ships is given in a deterministic manner where the requirements stated in absolute terms must be met to ensure ship survivability.

Naval ships are subjected to uncertainty in sea environments, structural configuration, material properties, and environmental and operating conditions [2-3]. While probabilistic methods have been applied extensively to quantify uncertainty of sea environments and estimate reliability of naval vessels under normal seaway loads, their extension to surface ships subjected to extreme dynamic loads due to collision, grounding, and weapon effects has not

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been well explored. Under these extreme loading environments, experimental observations have shown that the loading process is non-Gaussian and non-stationary and the ship structural response is nonlinear. The nonlinear structural response is induced by the initiation and evolution of multiple local damages, such as local plastic deformation, stiffener tripping, panel buckling, or fracture. The complexity in fluid-structure interaction phenomena will render the assumption on the loading process (i.e. stationary and Gaussian) invalid. The conventional approach [4] based on linear random vibration theories and peak statistics is inapplicable for the probabilistic vulnerability assessment of surface ships subjected to a hostile environment. Therefore, it is imperative to develop a generalized simulation based probabilistic analysis tool such that no limitation is placed on the nature of input random processes (Gaussian, non-Gaussian, stationary, or non-stationary) and system characteristics (linear or nonlinear).

The inverse first order reliability method (IFORM) has been developed and applied to the reliability-based design of a civil structure subjected to static loading [5]. However, its extension to a structural dynamic system subjected to an extreme environment is very limited. The application of the inverse reliability method to a surface ship subjected to a hostile environment is of crucial importance in terms of 1) achieving a balanced design between structural subsystems for a given threshold requirement; and 2) identifying a key design parameter for retaining a sufficient level of mission readiness after a hit.

The technical objectives of this research program are: 1) to assess the variance in vulnerability assessment due to known variability in material properties, geometric configuration, failure criteria, and loading parameters; 2) to develop a simulation based probabilistic analysis framework for a structural dynamics with random variables and random processes; 3) to develop an IFORM for a structural dynamic problem with random variables within the context of finite element reliability analysis; and 4) to apply the developed probabilistic analysis modules to a hull structure subjected to parametric dynamic loading. To achieve these research objectives, a general simulation based probabilistic framework (SIMLAB) is developed first for a nonlinear dynamic system by integrating 1) random variable generating modules; 2) random process simulation modules; and 3) user selected deterministic solver and limit state function. The great versatility of SIMLAB provides us a solid foundation for the development of more advanced probabilistic analysis tools such as IFORM for reliability-based ship design. The computational framework of IFORM is established by integrating a modified HL-RF search algorithm, the active gradient projection module, and a deterministic FEM solver (DYNA3D).

In this paper, we will summarize our technical approach in developing SIMLAB along with demonstration of SIMLAB via two numerical examples. A nonlinear structural dynamics code, DYNA3D, has been integrated into the probabilistic computational framework. A free-free beam with uncertainties in strength variables subjected to a random excitation is selected for tool validation and model exploration. The development and application of IFORM will be given in a forthcoming paper.

OVERVIEW OF A SIMULATION BASED PROBABILISTIC ANALYSIS FRAMEWORK (SIMLAB)

SIMLAB is a general probabilistic analysis framework which can be integrated with user provided solution modules to perform probabilistic response analysis and failure prediction of a nonlinear dynamic system subjected to a given random loading. As shown in Figure 1, both random sampling and Latin Hypercube sampling techniques are implemented in SIMLAB for generating random variables and random processes. Three solution modules, namely, an input update module, a deterministic solver, and a limit state function, are defined by a user for his own analysis. Some of these key components are described briefly in the subsequent sections.

RANDOM VARIABLES AND RANDOM PROCESS SIMULATION MODULES

All random variables with a given cumulative density function $F(X)$ (CDF) are generated based on a simulation of a uniformly distributed random variable $U \in [0,1]$ followed by an inverse CDF transformation, denoted by $F^{-1}(U)$. The CDF can be given either in an analytical form or in a table of pairs of data $((X, F(X))$. At present, the random simulation module in SIMLAB can generate multiple random variables with the following distributions: uniform, normal, lognormal, linear, arbitrary continuous, Gamma, shifted Beta, exponential, Weibull, and shifted Rayleigh.

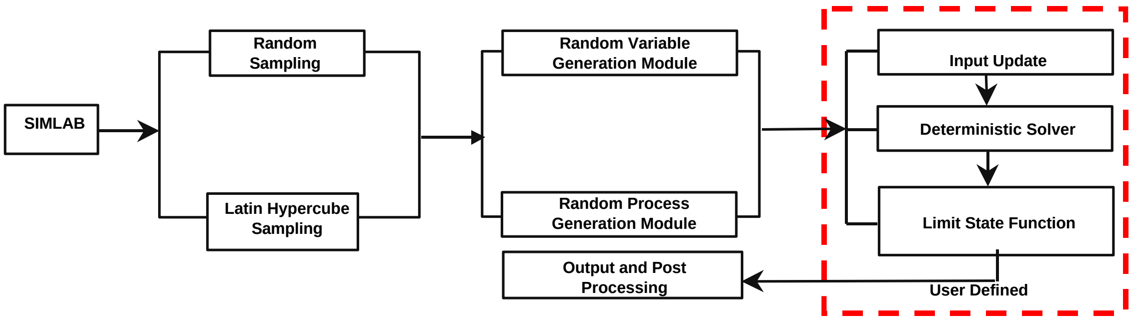

Figure 1. Display of Key Components in SIMPLAB

A random process simulation module has been developed to generate stationary Gaussian processes, non-stationary Gaussian processes, non-Gaussian stationary processes, and non-Gaussian non-stationary processes. For a Gaussian stationary process, the spectral representation method [6] is employed. Given a spectral density function of a Gaussian stationary random process ($g_{GS}(t)$), and a mean value of the process, $\bar{g}$, we will first divide the entire band of frequency $w \in [w_l, w_u]$ into N intervals and the *k-th* discrete frequency can be expressed as

$$w_k = w_l + (k-1)\Delta w \tag{1}$$

where $\Delta w = (w_u - w_l)/N$. The Gaussian stationary process can be simulated by

$$g_{GS}(t) = \bar{g} + \sum_{k=1}^{N+1} A_k \cos(w_k t + \Phi_k) \tag{2}$$

In Equation (2), A_k ($k=1, 2,\ldots, N+1$) are Rayleigh distributed random variables with mean and variance given by

$$\overline{A_k} = s_k \sqrt{\frac{p}{2}}; \qquad Var\{A_k\} = s_k^2 \left(2 - \frac{p}{2}\right) \tag{3}$$

where s_k is written as

$$s_k = \left[2S_g(w_k)\Delta w\right]^{1/2} \tag{4}$$

In Equation (4), S_g is the spectral density function of the random process. The random phase angle (Φ_k) given in Equation (2), is a uniformly distributed random variable in the interval [0, 2π]. To simulate a non-stationary process (g_{NS}) from a stationary process (g_S), a time-dependent deterministic function $u(t)$ is added to $g_S(t)$ as shown below:

$$g_{NS}(t) = g_S(t) + u(t) \tag{5a}$$

$$u(t) = A_0 \cos(w_0 t + f_0) \tag{5b}$$

where A_0, f_0, and w_0 are constants. Note that Equation (5a) represents the first order perturbation from a stationary process in the sense that only the first order moment (mean) is a time dependent function.

Two approaches have been used in SIMLAB to generate a non-Gaussian process (g_{NG}) from the corresponding Gaussian (g_G) process. In the first approach, we use a single parameter (random phase angle) based simulation model [7] along with a small number of frequency discretization points, N, i.e.,

$$g_{NG} = \sqrt{2} \sum_{k=1}^{N} s_k \cos\left(w_k t + \Phi_k\right) \tag{6}$$

It can be shown that g_{NG} will approach to a Gaussian process (g_G) as $N \to \infty$. The functional relation between the degree of non-normality and sampling rate N is given in Reference [8]. In the second approach, a nonlinear transformation is introduced to generate a non-Gaussian process from the corresponding Gaussian process [9], namely,

$$g_{NG} = G\left[g_G\right] \tag{7a}$$

$$G(x) = \frac{x + b\left(\mathrm{sgn}(x)\right)\left(|x|^n\right)}{C} \tag{7b}$$

where sgn(.) is the signum function [sgn(x)=1 for x>0, 0 for x=0 and -1 for x<0), and n (>0) and b(>0) are non-normality controlling parameters. The parameter C is determined in such a way that the derived non-Gaussian process has the same root-mean-square as the original Gaussian process. The expression for C can be found in Reference [9].

SIMULATION BASED PROBABILISITC ANALYSIS FRAMEWORK (SIMLAB)

The probabilistic simulation framework is formulated in a way that a deterministic solver can be easily integrated as a black box. The author has developed several probabilistic fatigue/fracture analysis tools by integrating a boundary element solver [10] and a finite element solver [11] into a computational framework. The similar approach has been used in the development of SIMLAB.

The key components of the SIMLAB methodology are shown in Figure 2. In a nested simulation, the outer loop is applied to simulate all random variables that are used to characterize basic structural strength variables (material properties and geometric parameters). The inner loop is used to simulate a random process (seaway/slamming loading). After selecting the material properties, hull geometric parameters, and applied loads, a finite element input file will be updated and a structural dynamic response analysis will be performed via a FEM solver. The maximum response variable over the entire loading period at a critical location will be stored and used in a limit state function. The first-excursion failure event is defined as the crossing of the critical response quantity above a safe threshold during the loading process. The final probability of failure will be calculated by the ratio of the total number of failures to the total number of simulations. By post-processing the simulation results, we can determine the most probable failure location, peak distributions, and extreme peak distributions.

EXAMPLE APPLICATION OF SIMLAB

To demonstrate the applicability of the developed simulation-based probabilistic analysis tool and validity of random process simulation modules, we consider a USCG Island Class Patrol Boat subjected to seaway loads as shown in Figure 3. A free-free beam subjected to a distributed dynamic load is used to characterize the structural response of the ship hull. A structural dynamics code, DYNA3D, is selected as the deterministic solver in SIMLAB. The safety margin for the *n-th* beam element is defined as

$$G_n(t, x_n) = s_y^n - s_{eff}^n(t, x_n) \qquad (8)$$

where s_y^n is the yield strength of the *n-th* beam element and $s_{eff}^n(t, x_n)$ is the VonMises stress of the *n-th* element at time *t*. The limit state function is given by

$$G = \min\left\{G_n(t, x_n)\right\}; \quad 1<n<M; \quad t \in [0, T] \qquad (9)$$

where M is the total number of beam elements and T is the termination time of response analysis. Note that the safety margin G_n defined by Equation (8) is quite simple. However, the resulting limit state function given by Equation (9) is highly nonlinear due to the operator of minimization used. Since the first order reliability method (FORM) is based on the linearized limit state function at a design point, the direct application of FORM will render the solution inaccurate. The simulation-based tool provides us a simple/direct way to calculate the first-excursion failure probability.

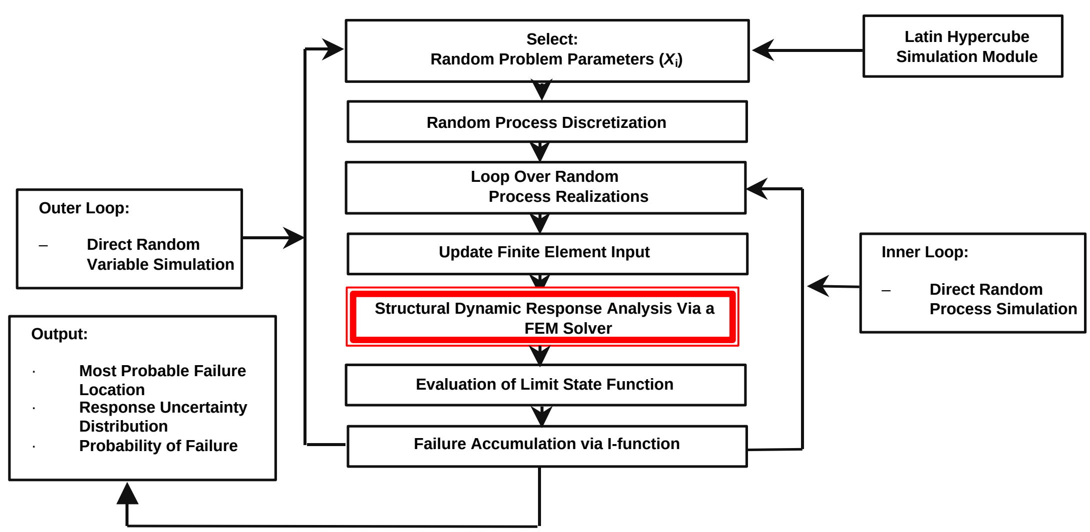

Figure 2. Overview of SIMLAB Methodology

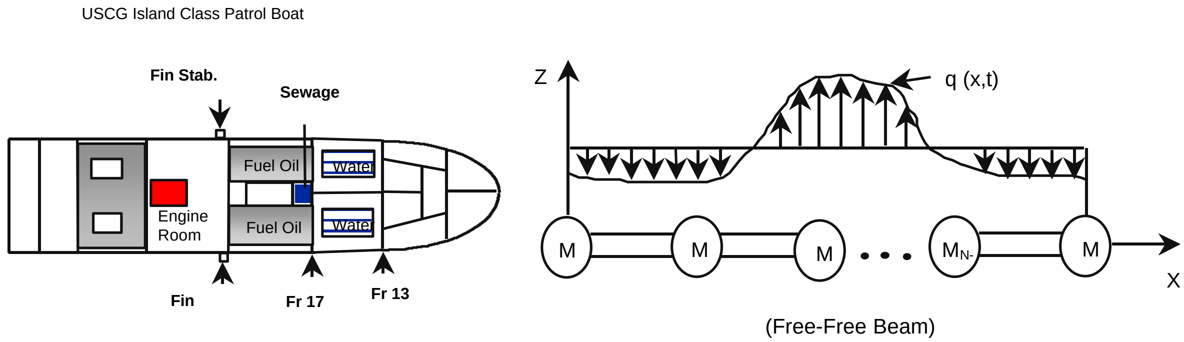

Figure 3. A Free-Free Beam Representation of a Ship Hull under Seaway Loading

The free-free beam model shown in Figure 3 is used to capture the primary hull girder response of a USCG Island Class Patrol Boat subject to a seaway loading. The hull girder is discretized into 24 beam elements. At each nodal point, a nodal mass is assigned to represent the sum of the structural mass (M_s), and the added mass (M_A). The structural mass consists of both the material mass (M_M) and the equipment mass (M_E). Based on Reference [12], the key deterministic parameters and mean values of random variables are listed in Table 1. The spatial distribution of nodal mass weighting factor ($w_i^m(x)$) and nodal pressure load ($p_0(x)$) are shown in Figs. 4a, and 4b, respectively.

Table 1. Deterministic Parameters and Mean Values of Random Variables

◆ **Problem Parameters**	
Length Overall..................	L_s = 33.51 m
Beam at Deck Amidships...	B = 6.42 m
Hull (Steel)…………	$\left(\bar{E} = 2.07\times10^{11}\ Pa,\ \ \bar{s}_y = 2.76\times10^{8}\ Pa\right)$
Added Mass (M_A)………………	$M_A \approx 1.25\ M_S = 1.25\left(M_E + M_M\right)$
Average Hull Thickness…….	$\bar{t}$ = 12.69 mm
Average Structural Mass……	$\bar{M}str = 353.89 kg$
Average Total Nodal Mass….	$\bar{M}^T_{node} = 577.21\ kg$
Average Pressure……………	$\bar{q}$ = 103425 Pa
◆ **Dynamic Pressure Load**……	$P(x,t) = p_o(x)\ f(t)$
◆ **Discrete Nodal Mass**………	$m_i = w_i^m \times M^T_{node}$

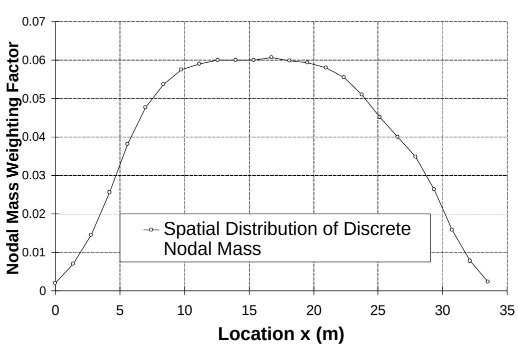

Figure 4a. Spatial Distribution of Nodal Mass Weighting Factor ($w_i^m(x)$)

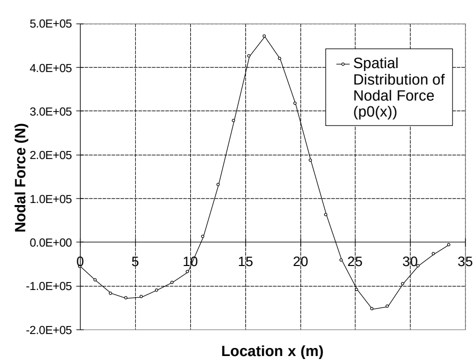

Figure 4b. Spatial Distribution of Nodal Force ($p_o(x)$)

In order to characterize the random temporal variation of the applied nodal force, a random process is used to describe the load amplification factor *f(t)* (see Table 1). The two-parameter Bretschneider's model [13], which has been used extensively for characterizing random seaway loading, is employed in this study. Based on the two-parameter Bretschneider's model, the mean square spectral density is given by

$$S(w) = \frac{5}{16}\left(\frac{w_m}{w}\right)^4 \frac{H_s^2}{w} e^{-1.25(w_m/w)^4} \quad (10)$$

where H_s is the significant wave height, and w_m is the wave frequency at the maximum wave height. Using $H_s = 5.0m$ and $w_m = 52.36$ *rps*, the resulting spectral density function is shown in Figure 5a. As shown in Figure 5a, the band width of *S(w)* is small compared with the magnitude of the center frequency. Thus, the process can be classified as a narrow band process. By using the above spectral density function *S(w)* in Eq. (4), the time histories of loading curves with a zero-mean can be generated as shown in Figure 5b.

In addition to the random process (see Figure 5b), a set of random variables are also used to characterize uncertainty in elastic modulus (E), yield strength (s_y), sectional thickness (t), and total nodal mass (m_{node}^T). The statistical models along with model parameters for these random variables are listed in Table 2. The coefficient of variation (COV), which is a measure of the degree of scatter of a random variable, is adopted based on the published information (see Table 2).

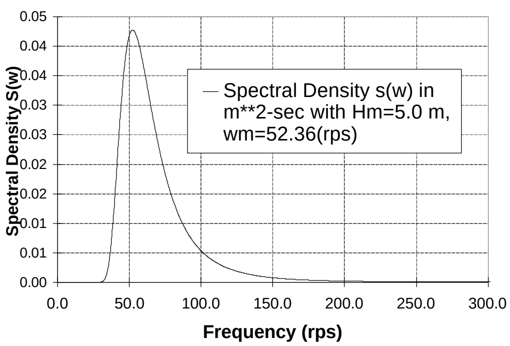

Figure 5a. Mean Square Spectral Density of Load Amplification Factor f(t)

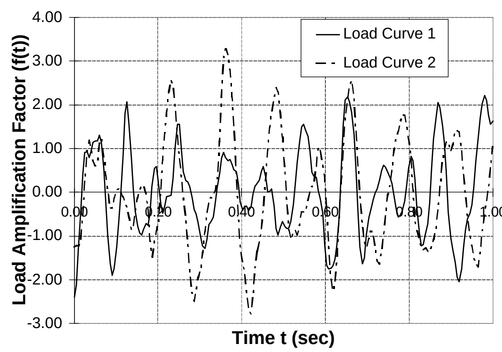

Figure 5b. Two Loading Curves Generated from the Gaussian-Stationary Random Process

Table 2. Statistical Models of Input Random Variables

Random Variables	**Distribution**	**Mean**	**Coefficient of Variation (COV)**
Sectional Young's Modulus E (Pa)	LOG NORMAL	2.07×10^{11}	3% (SSC-351) Reference [14]
Sectional Yield Strength s_y (Pa)	LOG NORMAL	2.76×10^{8}	10% (SSC-351) Reference [14]
Sectional Thickness t (mm)	LOG NORMAL	12.69	3.8% (White et al. 1995) Reference [2]
Total Nodal Mass m_{node}^T (lbm)	UNIFORM	577.21	15%

RESULTS AND DISCUSSIONS

In order to demonstrate the validity and accuracy of the developed simulation based probabilistic analysis tool (SIMLAB) for the response analysis of a uncertain dynamic structure, an elastic beam subjected to a stationary narrow band Gaussian process (see Figs. 5a and 5b) is considered first. For a linear dynamic system subjected to a stationary narrow band Gaussian excitation, the statistical distributions of response and extreme peak values can be described by a Rayleigh and a Gumbel distribution, respectively [14]. These analytical peak statistical distributions have been used extensively for the present reliability-based ship design [2, 14].

Before the probabilistic analysis, a deterministic analysis is performed first using mean values of all random variables (see Table 1) and load curve 1 (see Figure 5b). The stress is found to be maximum at the mid-span of the beam (i.e. Element 13). The time history of the beam axial stress at element 13 is given in Figure 6. Both positive (negative) peak and the extreme peak values are also shown in Figure 6. As can be seen from Figure 6, the peak values in axial beam stress for *t>0.6 sec* exceed the mean value of yield strength ($\bar{s}_y = 2.76E+08\ Pa$). However, with the use of elastic beam model, the effect of material nonlinearity on the response uncertainty cannot be characterized.

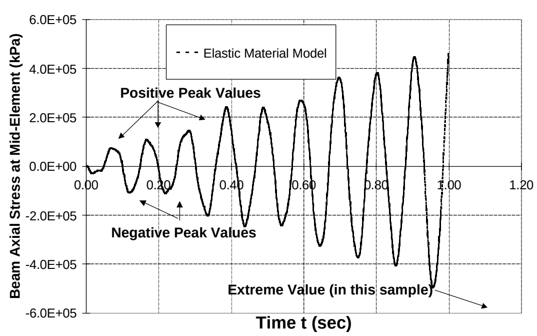

Figure 6. Time History of Beam Axial Stress at Element 13

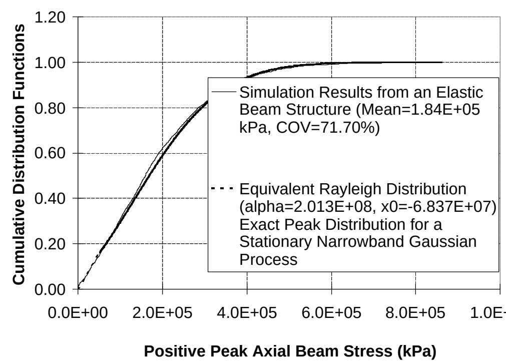

Figure 7. Statistical Distribution of Positive Peak Values for the Elastic Beam Model

In the probabilistic analysis, these positive (negative) peak values are stored during the simulation and the extreme peak value is searched for a given response time history (see Figure 6). These data are then used in a post-statistical processing module within SIMLAB to determine their statistical distributions. In order to quantify the response uncertainty, 1000 simulations are performed using SIMLAB. The statistical distribution of positive peak values for the present elastic beam model is shown in Figure 7 along with its comparison with an equivalent Rayleigh distribution. The two parameters used in Rayleigh distribution $(a = 2.013E + 05kPa, x_0 = -0.683E + 05kPa)$ are determined by equating the first two statistical moments (mean and variance) of the Rayleigh distribution to the simulated statistical moments. As shown in Figure 8, the statistical distribution of positive peaks for the elastic beam model agrees very well with the Rayleigh distribution. This excellent agreement also holds for the statistical distribution of extreme peak values (see Figure 8). The two parameters used in Gumbel distribution as shown in Figure 8 $(u_n = 2.5E + 05kPa, a_n = 0.95E - 08m^2 / N)$ are also determined by fitting the first two statistical moments the Gumbel statistical model to the moments of simulated data. The consistency between the numerical simulation results and analytical statistical models for a linear dynamic system subjected to a stationary Gaussian process validates the applicability and accuracy of the developed simulation based probabilistic analysis tool (SIMLAB).

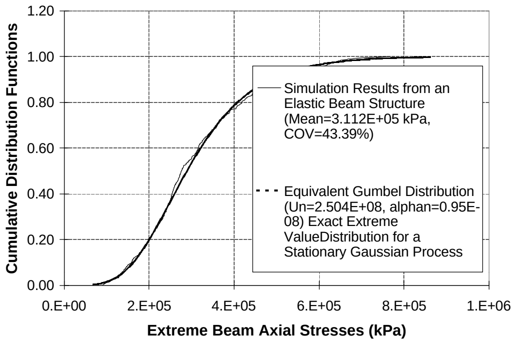

Figure 8. Statistical Distribution of Extreme Peak Values for the Elastic Beam Model

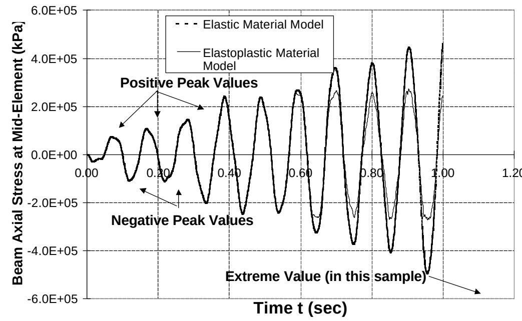

Figure 9. Comparison of Time Histories of Beam Axial Stress Using Linear and Nonlinear Material Models

In order to investigate the effect of material nonlinearity on the statistical distributions of peak and extreme values, an elastoplastic beam structure subjected to a stationary Gaussian process (see Figure 5b) is considered here. In addition to other random variables used in the previous example, the yield strength of the steel hull is assumed as a random variable with a lognormal distribution (see Table 2). The failure event is defined as the crossing of a critical response quantity (the maximum VonMises stress) above a safe threshold (yield strength) during the process of the dynamic loading (see Equation 8). A deterministic analysis using mean values of all random variables and load curve 1 (see Figure 5b) is performed first and the time history of the beam axial stress in beam element 13 is shown in Figure 9. Unlike the elastic beam model, the beam deforms plastically after *t=0.6 sec*. The resulting maximum stress is bounded by the yield strength ($s_y = 2.76 \times 10^8 \, Pa$).

One thousand (1000) simulations are performed in the probabilistic analysis. Using the limit state function defined by Equation (9), the total number of the first-excursion failure events among 1000 simulations is 448 and the resulting probability of failure is 0.448. The statistical distributions of both the critical response parameter (maximum VonMises stress) and the strength parameter (yield strength) are shown in Figure 10. The failure probability can be directly obtained from the CDF of the VonMises stress if the yield strength is a deterministic parameter. For instance, by using the mean value of the yield strength ($s_y = \bar{s}_y = 2.76 \times 10^5 \, kPa$), the resulting failure probability is 0.27 (1-CDF (2.76E+05)). Thus, we can conclude that the introduction of uncertainty in yield strength will increase the failure probability from 0.27 to 0.448.

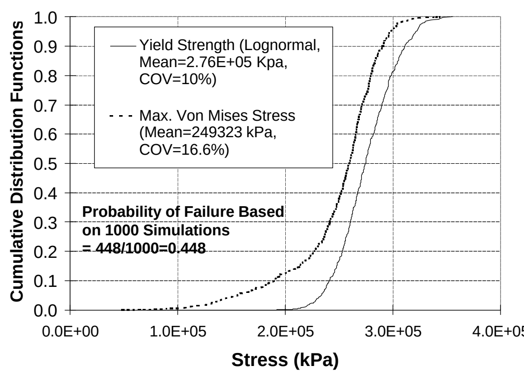

Figure 10. Comparison of Cumulative Distribution Functions of Maximum VonMises Stress and Yield Strength

The comparison of CDFs of positive peak values with an equivalent Rayleigh distribution is shown in Figure 11. Unlike the case of linear material model, the statistical distribution of positive peak values has a large deviation from the equivalent Rayleigh distribution despite that two distributions have the same mean and variance. Similarly, the statistical distribution of extreme peak values can no longer be described by an equivalent Gumbel distribution (see Figure 12).

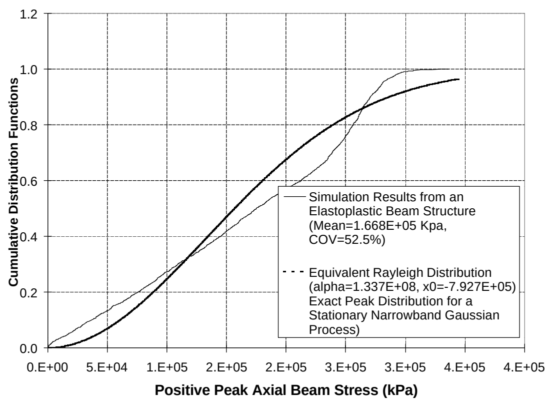

Figure 11. Statistical Distribution of Positive Peak Values for the Elastoplastic Beam Model

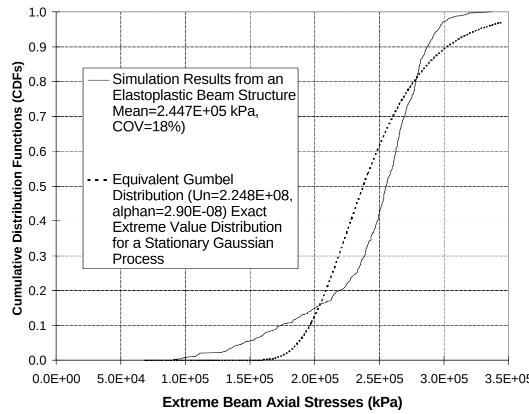

Figure 12. Statistical Distribution of Extreme Peak Values for the Elastoplastic Beam Model

A common feature can be observed from Figures 11 and 12. In both the lower and the upper tail regions of these CDF curves, the CDF obtained from the simulated response data is larger than the one predicted from the corresponding analytical model, which is valid for a linear dynamic system subjected to a Gaussian process. Since the small failure probability of a structural system is governed by the upper tail CDF curve of the critical response parameter, we may conclude that the use of these analytical peak distributions for a nonlinear system may result in a conservative assessment of failure probability.

CONCLUSIONS

In this study, we have shown the successful integration of the DYNA3D code with the developed simulation-based probabilistic analysis framework (SIMLAB). The validity and accuracy of SIMLAB have been demonstrated via its application to an elastic beam subjected to a random excitation. With the aid of an elastoplastic beam model, we can conclude that the material nonlinearity has a large impact on statistical distributions of key response parameters, their peak values and their extreme values. While the analytical statistical model of peak and extreme values fully capture the response statistics of a linear system subjected to a stationary Gaussian process, its extension to a nonlinear structural system may lead to a conservative assessment of failure probability. The effect of a non-stationary and non-Gaussian process on the response statistics and failure probability is currently under investigation. The great versatility of the simulation based probabilistic analysis framework (SIMLAB) provides us a solid foundation for the development of more advanced probabilistic analysis tools such as IFORM for reliability-based ship design.

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