SSC-180

Experimental Determination of Plastic Constraint Ahead of a Sharp Crack Under Plane-Strain Conditions

by

G. T. Hahn and A. R. Rosenfield

Ship Structure Committee

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ADDRESS CORRESPONDENCE TO:

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December 1966

Dear Sir:

The Ship Structure Committee has been sponsoring an experimental study at Battelle Memorial Institute of localized yielding around a notch. "Experimental Determination of Plastic Constraint Ahead of a Sharp Crack Under Plane-Strain Conditions" by G. T. Hahn and A. R. Rosenfield shows how the maximum normal stress generated in front of a crack can be deduced from experiments. This report is the third progress report on the project.

The Project has been conducted under the advisory guidance of the National Academy of Sciences-National Research Council, utilizing its Ship Hull Research Committee.

Comments on this report would be welcomed and should be addressed to the Secretary, Ship Structure Committee.

Sincerely yours,

John B. Oren

JOHN B. OREN
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

SSC-180

4th Progress Report
on
Project SR-164
"Local Strain Measurement"
to the
Ship Structure Committee

EXPERIMENTAL DETERMINATION OF PLASTIC CONSTRAINT AHEAD OF A SHARP CRACK UNDER PLANE-STRAIN CONDITIONS

by

G. T. Hahn and A. R. Rosenfield

Battelle Memorial Institute
Columbus, Ohio 43201

under

Department of the Navy
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Washington, D. C.
U. S. Coast Guard Headquarters
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ABSTRACT

An experimental method of indentifying the plastic constraint ahead of a sharp crack loaded under plane-strain conditions is proposed. The method is based on the idea that the cleavage stress--which can be measured with unnotched bars--is the peak stress developed ahead of a crack just prior to crack extension. Ways of calculating the strain, strain rate, and yield stress appropriate for the plastic region just ahead of the crack are developed. The ratio of the cleavage stress to the local yield stress indentifies the plastic constraint factor at the stress level corresponding to crack extension. Experimental results recently reported by Krafft are shown to be consistent with this interpretation. With these data, the following expression for p.c.f., the plastic constraint factor, is deduced: $\text{p.c.f.} = 1 + 2\,\frac{K}{Y}$, where Y is the yield stress, K the stress intensity parameter, and the numerical constant, 2, has the dimensions inches $^{-1/2}$. This result offers a way of formulating K_{Ic}, the fracture toughness for crack extension by cleavage, in more basic terms and sheds some light on the metallurgical origins of K_{Ic}.

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INTRODUCTION

A crucial but unresolved feature of the crack extension problem is the stress intensification and triaxiality existing ahead of a sharp crack loaded under plane-strain conditions. Elastic stress-concentration factors are not meaningful once the peak stress exceeds the yield stress (for the case of a sharp crack, this occurs at very low nominal stress levels) because further stress build-up is cancelled by local plastic deformation, at least initially. However, the plastic zone must be accommodated within an elastic matrix. This imposes a continuity requirement difficult to satisfy under plane-strain conditions, and is responsible for "plastic constraint": triaxiality and stress intensification above the level of the yield stress.[1]

Stress intensification after the onset of localized yielding is usually expressed by a plastic constraint factor (p.c.f.):

$$\text{p.c.f.} \equiv \frac{\sigma_{max}}{Y} \tag{1}$$

where σ_{max} is the maximum normal stress and Y is the yield stress. There is an upper limit for the constraint that can be estimated from slip-line field theory: p.c.f. = 2.57.† [1] This value corresponds to a plastic zone size comparable to the dimensions of the sample (a condition referred to as full plasticity or general yielding), and is not at issue. The essential problem is assigning p.c.f. values for intermediate zone sizes: after the onset of local yielding, but before full plasticity. So far, no calculations have been reported for a sharp crack but Hendrickson, Wood, and Clark[2] and Barton and Hall[3] have calculated p.c.f.'s for reasonably sharp hyperbolic notches under plane strain (see Figure 1). The initial portions of their curves: $0 < \frac{\sigma_{max}}{Y} < 1$, reflect elastic behavior (the

† This is based on a Tresca yield criterion. The corresponding value for a Von Mises yield criterion is p.c.f. = 2.82.

slopes of $\overline{AB}$ and $\overline{AB}'$ are the elastic stress concentrations); in the plastic region: $\frac{\sigma_{max}}{Y} > 1$, the p.c.f. rises gradually with T/Y, where T is defined as the nominal stress. For a sharp crack, one with a root radius approaching zero, the slope of $\overline{AB}''$ approaches infinity, but beyond this the variation of the p.c.f. is not defined. The prevailing view seems to be that the constraint factor for a sharp crack increases much more rapidly with stress level (see dashed line in Figure 1).

This paper describes an experimental method for measuring the plastic constraint based on a special circumstance, namely: crack extension by cleavage of mild steel at low temperatures obeys a maximum stress criterion. Experimental results recently reported by Krafft[4] are analyzed on this basis. The analysis indicates that the p.c.f. for a sharp crack increases gradually, according to the relation p.c.f. = $1 + 2\left(\frac{K}{Y}\right)$, where Y is the yield stress, K is the stress intensity parameter, and the numerical constant, 2, has the dimensions inches$^{-1/2}$. The result is similar to that calculated for the hyperbolic notches. Implications with respect to the origins of cleavage fracture toughness are discussed.

METHOD OF ANALYSIS

The method of measuring plastic constraint, proposed here, takes advantage of certain special properties of mild steel:

1. Cleavage Stress. The cleavage of mild steel occurs at a relatively constant, reproducible value of stress,[5-7] symbolized here by σ^*_{cleav}, which can be measured by breaking unnotched bars at low temperature. Both theory (see Appendix A) and experiments[8] agree that σ^*_{cleav} is substantially independent of temperature and strain rate.

In the case of a notched sample, cleavage cracks will be initiated when $\sigma^*_{max} = \sigma^*_{cleav}$. Since σ_{max} is attained at a point close to the elastic-plastic boundary,[2,3] cleavage will tend to occur first in a region that has undergone little prior strain. This means that the value of σ^*_{cleav} corresponding

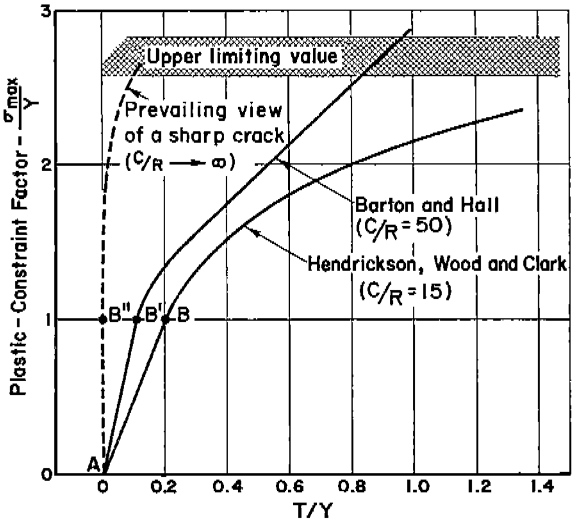

Fig. 1. Influence of Nominal Stress Level on the Plastic-Constraint Factor. Ratio of Half-Crack-Length (c) to Crack Tip Radius (r) are indicated.

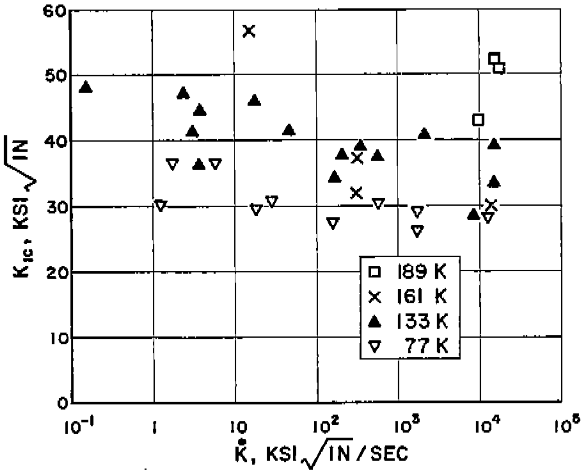

Fig. 2. Influence of Temperature and Loading Rate on the Fracture Toughness of Mild Steel According to Krafft.

to the virgin material is appropriate and the strain dependence of σ^*_{cleav} can be ignored.[9]

2. Crack Extension by Cleavage. The initiation of cleavage cracks close to the elastic-plastic boundary in a sharply notched or pre-cracked sample is not likely to be self-limiting. The presence of cracks will not relieve the state of triaxiality or lower σ_{max}, while the high speed with which cleavage cracks grow will enhance the local value of Y. For these reasons, cleavage initiation is expected to trigger unstable crack extension and fracture; in fact, a slow growth stage is not observed.[10] For these special circumstances, the value of the constraint factor can be calculated from σ^*_{cleav}:

$$\left.\begin{array}{l} \sigma_{max} = \sigma^*_{cleav} \\ \\ \text{p.c.f.} = \dfrac{\sigma^*_{cleav}}{Y_{\theta,\dot{\epsilon}}} \\ \\ \end{array}\right\}_{K = K_{Ic}} , \qquad \begin{array}{r} (2) \\ \\ (3) \end{array}$$

where $K = K_{Ic}$ signifies that the equations are only valid just prior to crack extension and $Y_{\theta,\dot{\epsilon}}$ must be the yield stress at the temperature and strain rate existing at the crack tip.

3. The Yield Stress of Mild Steel. At the temperatures and strain rates that favor cleavage fracture, the yield stress of mild steel is strongly temperature and strain-rate dependent. These dependences can be expressed by the following empirical equation valid for fine-grained mild steels in the range 250 K $< \theta <$ 77 K and 10^{-4} per sec $< \dot{\epsilon} < 10^{+3}$ per sec:[9]

$$Y(\text{psi}) = Y_s(\text{psi}) + 195{,}000 - 11{,}100\,\theta^{1/2}\ (^\circ\ \text{Kelvin}) + 8{,}000 \log \dot{\epsilon}\ (\text{per sec}) \quad , \tag{4}$$

where Y is the yield stress at the given θ (temperature) and $\dot{\epsilon}$ (strain rate), and Y_s is the yield stress at room temperature for $\dot{\epsilon} = 10^{-3}$ per sec. The strain rate at the tip of a sharp crack can be estimated from the following equation derived in Appendix B:

$$\dot{\epsilon} \approx \frac{1}{\ell E} \left(\frac{K}{Y_{\theta,\dot{\epsilon}}}\right) \dot{K} \quad , \tag{5}$$

where E is the modulus, $K \equiv T\sqrt{\pi c}$, $\dot{K} \equiv \dot{T}\sqrt{\pi c}$, $\dot{T}$ the nominal stressing rate, c is either the length of an edge crack or the half length of a center crack, and $\ell \approx 0.001$ in. is the extent of the plastic zone ahead of the crack.

Together, Equations (4) and (5) describe the yield stress appropriate for the plastic zone ahead of the crack. In principle, the influence of strain hardening on the yield (or flow) stress must also be taken into account, but this is beyond the competence of the present treatment. It should be noted here that the strain-hardening contribution is relatively small at temperatures close to 77 K, the testing range exploited in this paper.

Plastic constraint can therefore be deduced from tests of pre-cracked plates or bars of a mild steel whose σ^*_{cleav} and yield characteristics are known. The tests must be performed at the relatively low temperatures and/or high loading rates that favor crack extension exclusively by cleavage.† By varying the test temperature and loading rate, the value of Y in Equation (3) can be

† Instances where cleavage is preceded by fibrous fracture require an analysis more sophisticated than the one offered here.

systematically altered. Since the fracture stress is also modified in this way, the relation between p.c.f. and stress level can be mapped out. The method is suitable for studying constraint in sheets or plates at the stress levels consistent with plane strain relaxation:[11]

$$\left(\frac{K_{Ic}}{Y}\right)^2 < \frac{8\ t}{\pi} \quad , \tag{6}$$

where t is the plate thickness. Stress levels that exceed this limit favor: (a) deformation through the plate thickness (plane stress), (b) the loss of constraint, and (c) a decreasing value of σ_{max}--circumstances that preclude cleavage.

Krafft[4] has recently reported tests of pre-cracked mild steel plates that lend themselves to a p.c.f. analysis. His results, summarized in Figure 2, were obtained on 1/4-in. thick single-edge fatigue cracked plates machined from line-pipe steel (API grade 5L-X52, C: 0.24%, Mn: 1.10%, Si: 0.01%, P: 0.012%, and S: 0.020%): grain diameter = 0.012 mm,† Y_s = 52,600 psi, σ^*_{cleav} = 200,000 psi.‡ Reported values of $\dot{K}$ and K_{Ic} were converted into $\dot{\epsilon}$ values with the aid of Equation (5), and these were used to calculate Y with Equation (4). [This Equation (4) offers an excellent description of the yield characteristics of this particular steel.[9]]

RESULTS AND DISCUSSION

Calculated plastic constraint values are plotted against stress intensity expressed as $\frac{K}{Y}$ (really $\frac{K_{Ic}}{Y}$ in these experiments) in Figure 3. Comparison with Figure 2 shows that systematic changes with temperature and strain

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† Measured at Battelle.

‡ An X-52 steel having practically the same composition and grain size (C: 0.26%, Mn: 1.15%, grain diameter: 0.012 mm) has been tested at Battelle.[9] Unnotched bars fractured by cleavage at 77 K at a strain rate of 10^2 per sec after straining, 13%. The yield stress under these conditions is 185,000 psi, the true-fracture stress 215,000 psi, and the σ^*_{cleav} for unstrained material somewhere in between. On this basis a value $\sigma^*_{cleav} \doteq$ 200,000 psi is a reasonable estimate, probably accurate to within 5%.

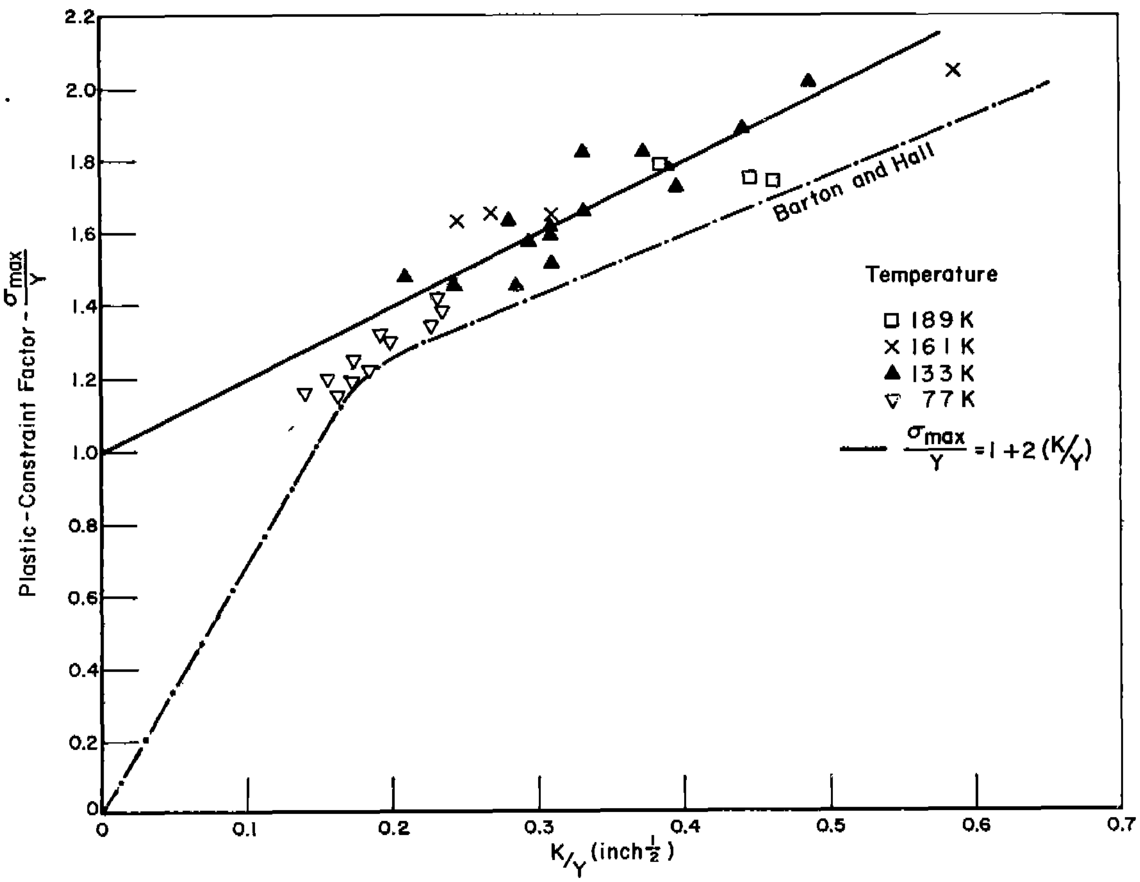

Fig. 3. Effect of Relative Stress Intensity on the Plastic-Constraint Factor of a Sharp Crack.

rate have all but disappeared, with normal test-to-test scatter the only variation remaining. It should be noted that this result is obtained in spite of drastic changes of the yield stress; values employed in the calculations ranged from 95,000 psi to 170,000 psi. The correlation is expressed by the following relation:†

$$\text{p.c.f.} = 1 + 2.0\left(\frac{K}{Y}\right) \quad . \tag{7}$$

The slight divergence from this trend evident when $\frac{K}{Y} < 0.2\sqrt{\text{in.}}$ is thought to be related to the relatively small zone size existing at these stress levels. One

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† Note that the ratio $\frac{K}{Y}$ is <u>not</u> dimensionless and that the factor 2.0 is in units of $(\text{in.})^{-1/2}$.

possibility is that the cleavage process begins to feel effects of the plastic zone that was introduced earlier when the samples were precracked by fatiguing at room temperature: e.g., local strain generated in this way can enhance the cleavage stress.[9] Another possibility is that the small number of grains (or portions of grains) highly stressed does not include grains that cleave easily (a cleavage stress-size effect). In either case the p.c.f. will be underestimated.

The implication of the present findings is that the constraint factor for a sharp crack is given by Equation (7), not only for steel but for other materials as well since the elastic properties do not influence constraint, at least to a first approximation.[1] The correlation displays three features that support the interpretation:

1. Substantial changes in K_{Ic} occasioned by different test temperatures and loading rates involving drastic changes in Y are reduced to a single correlation.

2. The correlation extrapolates to p.c.f. = 1, when $\frac{K}{Y} = 0$, consistent with expectations for a sharp crack.

3. The variation of p.c.f. with stress level is very similar to that calculated by Barton and Hall[3] at the higher stress levels, e.g., $\frac{K}{Y} > 0.2\sqrt{in.}$† (See Figure 3.) This is to be expected since the relatively small volume of material that must be added to convert a hyperbolic notch into a sharp crack will not alter constraints radically once the plastic zone is larger than the root radius.

At the same time, it must be noted that the present findings depart from the prevailing view of a p.c.f. that rapidly approaches the upper limiting value as pictured in Figure 1. The basis for this view needs to be re-examined. Also, more experimental evidence, perhaps derived from other steels and metals

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† Barton and Hall's[3] calculations are for 0.5-in. deep hyperbolic notches. These values of T/Y can be converted to approximate $\frac{K}{Y}$ values: $\frac{K}{Y} = \frac{T\sqrt{\pi c}}{Y}$ by the equivalent $c \approx 0.5$ in.

that undergo cleavage, is needed to buttress the correlation offered here.

In the meantime, it is useful to note some of the implications with respect to the fracture toughness when crack extension is exclusively by cleavage. Equation (7) can also be written:

$$\frac{\sigma^*_{cleav}}{Y} = 1 + 2\left(\frac{K_{Ic}}{Y}\right) \tag{7a}$$

or

$$K_{Ic} = 1/2\,(\sigma^*_{cleav} - Y_{\theta,\dot{\epsilon}}) \quad , \tag{8}$$

and this is valid when plane strain prevails, e.g., $K_{Ic} < \sqrt{8Yt/\pi}$ (t is the plate thickness). Under these conditions, raising the cleavage stress relative to the yield stress improves the toughness. Crack extension by cleavage is not possible when $\sigma^*_{cleav} > \sim 2.7\ Y_{\theta,\dot{\epsilon}}$, irrespective of plate thickness, since the value of p.c.f. cannot exceed $\sim$ 2.7. However, it must be remembered that $Y_{\theta,\dot{\epsilon}}$ refers to the yield stress of the material at the temperature and <u>strain rate</u> developed at the crack tip, and this can be substantially greater than the value obtained from standard tensile tests. At low stress levels complications associated with precracking or a size effect may come into play, and the extent of this could depend on microstructure, the material, and the fatigue stress intensity. Finally, it should be noted that a good deal is known about the effects of a variety of metallurgical factors on the σ^*_{cleav} and Y of mild steel (see Reference 9 for a review), and this knowledge can now be translated into K_{Ic} predictions with the aid of Equation (8).

CONCLUSIONS

1. There are special circumstances when the peak stress ahead of a sharp crack can be identified with the cleavage stress derived from unnotched bars: (a) when localized yielding is predominatnly plane strain and (b) when the mode of crack extension is by cleavage.

2. With this interpretation the plastic constraint factor can be deduced from crack extension experiments; the relation p.c.f. = $1 + 2\left(\frac{K}{Y}\right)$ is derived from the measurements of Krafft.

3. The results obtained indicate the K_{Ic} for the cleavage mode can be expressed in terms of unnotched strength values: $K_{Ic} = 0.5(\sigma^*_{cleav} - Y)$, except possibly for very brittle conditions: $\frac{K_{Ic}}{Y} < 0.2$, where the interpretation may be complicated by prior fatiguing or by a cleavage stress-size effect.

ACKNOWLEDGMENTS

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REFERENCES

(1) Hill, R., The Mathematical Theory of Plasticity, Oxford University Press, London (1956), p 245.

(2) Hendrickson, J. A., Wood, D. S., and Clark, D. S., Trans. ASM, 50, 656 (1958).

(3) Barton, F. W., and Hall, W. J., "A Study of Brittle Fracture Initiation in Mild Steel", Ship Structure Committee Report SSC-147, 1963.

(4) Krafft, J., NRL Review of Progress, 1966.

(5) Hahn, G. T., Averbach, B. L., Owen, W. S., and Cohen, Morris, Fracture, J. Wiley, New York (1959), p 91.

(6) Hahn, G. T., Averbach, B. L., and Cohen, Morris, J. Iron and Steel Inst., 200, 634 (1962).

(7) Knott, J. F., and Cottrell, A. H., J. Iron and Steel Inst., 201, 249 (1963).

(8) Knott, J. F., J. Iron and Steel Inst., 204, 104 (1966).

(9) Rosenfield, A. R., and Hahn, G. T., "Numerical Descriptions of the Ambient, Low-Temperature and High-Strain Rate Flow and Fracture Behavior of Plain Carbon Steel" (to be published).

(10) Krafft, J. (private communication).

(11) Hahn, G. T., and Rosenfield, A. R., Acta Met., 13, 293 (1965).

(12) Hahn, G. T., and Rosenfield, A. R., "A Modified Double-Pile-Up Treatment of the Influence of Grain Size and Dispersed Particles on Brittle Fracture", Acta Met. (to be published).

(13) Hayward, E. R., and Greenough, A. P., J. Inst. Met., 88, 217 (1959-1960).

(14) Udin, H., Trans. AIME, 191, 63 (1951).

(15) Funk, E. R., Udin, H., and Wulff, J., Trans. AIME, 191, 1206 (1951).

(16) Lord, A. E., Jr., and Besher, D. N., J. Appl. Phys., 36, 1620 (1965).

(17) Dugdale, D. S., J. Mech. Phys. Solids, 8, 100 (1960).

(18) Rosenfield, A. R., Dai, P. K., and Hahn, G. T., Proceedings of the International Conference on Fracture, Sendai, Japan, 1965.

(19) Bilby, B. A., and Swinden, K. H., Proc. Roy. Soc., Series A, 285, 22 (1965).

APPENDIX A

TEMPERATURE DEPENDENCE OF THE CLEAVAGE STRESS

Experimental evidence indicates that the cleavage stress is, at most, only slightly temperature dependent. For example, Knott's[8] experiments in the range 123-173 K display a total dependence of about -5 percent per 100 K in one case and less than -1 percent in the second. In view of the fact that corrections as large as ± 10 percent per 100 K do not change the present results significantly, no corrections are applied to the data in Figure 3.

This lack of temperature dependence of the cleavage stress is consistent with theory. It is customary to represent the cleavage process by an idealized model such as a simple pile-up. Consequently, two temperature corrections must be considered. The first is derived from the model itself which yields a slightly temperature dependent cleavage stress, about -4 percent per 100 K in the range 77-300 K. The origin of the first correction is described in detail in the following paragraph. When the correction is applied to the calculations presented in Figure 3, the result is not altered significantly. Secondly, the degree to which the model approximates the real array may be temperature dependent, but this correction cannot be evaluated theoretically.

The authors[12] have recently modified the Cottrell treatment of the double pile-up model of cleavage, and this provides a basis for estimating the first correction. The various equations in Reference 12 can be reduced to the following formula for the cleavage stress:

$$\sigma^*_{\mathrm{cleav}} = \frac{4\,\gamma}{L}\left(\frac{D'}{\tau^*}\right)^{1.56} \quad . \qquad \text{(A-1)}$$

The quantity L, the length of dislocation arrays is governed by the microstructure and can be regarded temperature insensitive. The term γ is the surface energy: $\gamma \approx 2000$ ergs/cm^2 for iron. Measurements at high temperatures[13-15] indicate surface energy temperature dependence of about -50 ergs/cm^2 per 100 K or about -2.5 percent per 100 K. At low temperatures this dependence is likely to be smaller and closer to the shear modulus temperature dependence, about -1 percent per 100 K in the range 300-77 K.(16) The quantity D' is a function of the stiffness coefficients:

$$D' = \frac{1}{\pi\sqrt{2}} \left\{ 0.25\ C_{44} + 0.50\ (C_{11} + C_{12}) \left[\frac{C_{44}(C_{11} - C_{12})}{C_{11}(C_{11} + C_{12} + 2C_{44})} \right]^{1/2} \right\} \quad \text{(A-2)}$$

and displays a temperature dependence of -1.2 percent per 100 K [equivalent to -1.9 percent per 100 K for $(D')^{1.56}$] in the range 300-77 K.(16) The effective shear stress, τ^*, is formulated:

$$\tau^* = \tau_o - \tau \quad , \quad \text{(A-3)}$$

where τ_o and τ are the applied shear stresses identified with $\bar{V}_o$, the average dislocation velocity, and $\bar{V}$ a fixed lower value related to the average velocity of dislocations in pile-ups. Since velocities are proportional to strain rates: $\dot{\epsilon}_1/\dot{\epsilon}_2 = \bar{V}_o/\bar{V}$, and since $\tau = \frac{m}{2} \ln \dot{\epsilon}$ in the range 200-77,(9) the effective shear stress is:

$$\tau^* = \frac{m}{2} \ln \frac{\bar{V}_o}{\bar{V}} \quad . \quad \text{(A-4)}$$

Experimental results for Krafft's steel show that m is independent of temperature to within ± 3 percent in the range 200-77 K.(9) While there is not enough data to support a systematic variation in this temperature range a slight change, about 2 percent per 100 K is possible, and this would reinforce rather than cancel the temperature effects on γ and D'. Taken together, a cleavage stress correction of from -3 to -6.5 percent per 100 K is indicated.

APPENDIX B

THE STRAIN RATE IN THE PLASTIC ZONE AHEAD OF A SHARP CRACK

Solutions for the strains and strain rates generated ahead of a sharp crack during loading have not yet been worked out. However, displacement values have been calculated, and when these are combined with the dimensions of the plastic zone, then strain and strain rate values can be estimated. For example, at low stress levels, V_c, one-half the crack tip displacement and ρ, the plastic zone length for the DM† model[11,17,18] (see Figure B-1a) are:

$$V_c = \frac{\pi c Y}{2E}\left(\frac{T}{Y}\right)^2 \quad \text{or} \tag{B-1}$$

$$V_c = \frac{Y}{2E}\left(\frac{K}{Y}\right)^2 \tag{B-1a}$$

$$\rho = \frac{\pi^2 c}{8}\left(\frac{T}{Y}\right)^2 \quad \text{or} \tag{B-2}$$

$$\rho = \frac{\pi}{8}\left(\frac{K}{Y}\right)^2 \tag{B-2a}$$

where $K \cong T\sqrt{\pi c}$ is the Irwin stress intensity parameter, T is the nominal stress, c the crack half-length, Y the yield stress, and E is Young's modulus.

While the DM model is only meaningful for plane-stress conditions, e.g., thin sheets, Bilby and Swinden[19] report that values for a crack under plane strain with relaxation confined to two 45 degree-inclined slip planes (see Figure B-1b) take the same form:

$$V_{c\,(\text{plane strain } 45^\circ \text{ relaxed})} = 1/2\, V_{c\,(\text{DM})} \tag{B-3}$$

$$a_{(\text{plane strain } 45^\circ \text{ relaxed})} = a_{(\text{DM})} \tag{B-4}$$

or

$$\rho_{(\text{plane strain } 45^\circ \text{ relaxed})} = 1.4\, \rho_{(\text{DM})} \;. \tag{B-4a}$$

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† Dugdale-Muskhelishvili

Plastic zones revealed by etching (see Figure B-2) show that real plastic zones display an inclination closer to 75 degrees (Figure B-1c), and are not confined to a single slip plane but spread out (Figure B-1d). One would not expect that these features would alter V_c or ρ radically, and, in fact, plane-strain-type plastic zones revealed by etching can be approximated by the relation:(11)

$$\rho_{\text{(plane strain)}} \approx 1/2\, \rho_{\text{(DM)}} \quad , \qquad \text{(B-5)}$$

and this is supported by the data in Table B-1. Although a factor of 3 discrepancy between theory (Equation B-4) and experiment (Equation B-5) is evident, the zone size measurements do support the idea of a close relation between the DM

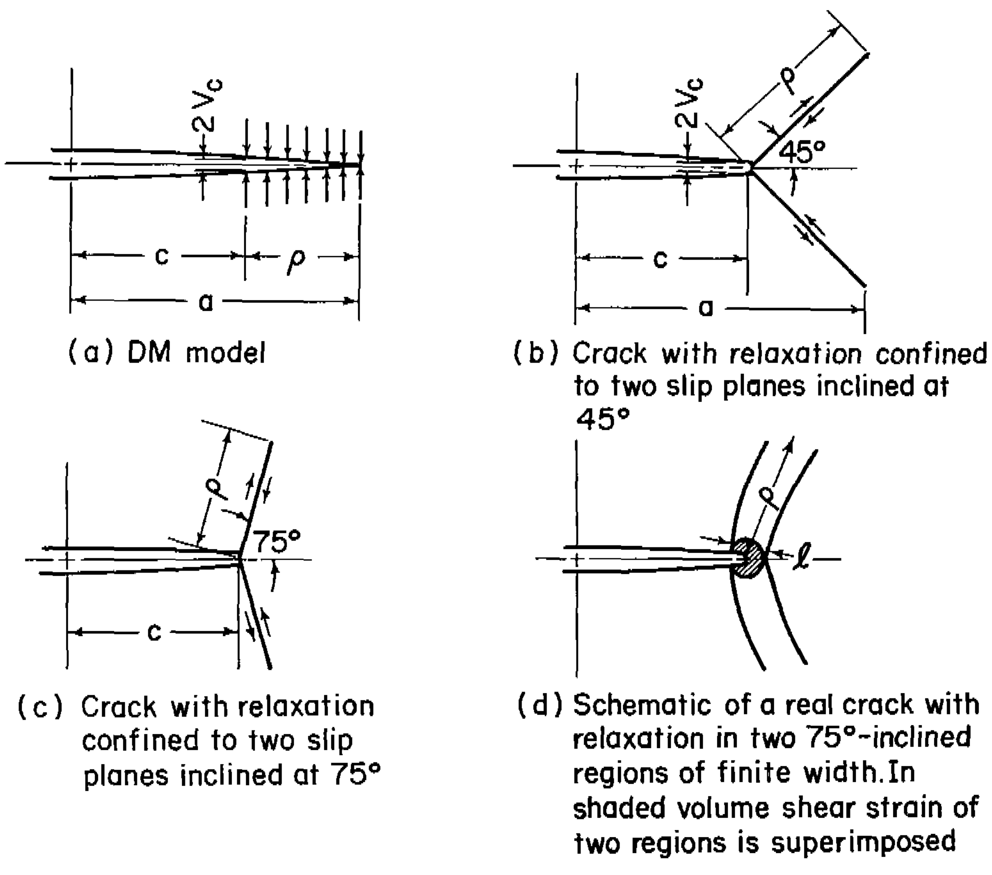

Fig. B-1. Models of a Crack with a Plastic Zone.
(a) Model of relaxation under plane stress
(b), (c), and (d) Models valid under plane strain

Fig. B-2. Plastic Zone Generated in a Fatigue-Cracked Fe-3Si Plate and Revealed by Etching.

The plate thickness is 0.060 in., K/Y = 0.266 (see Table B-1 for more details).

Magnification X180.

model and real plane-strain zones. Consequently, the following relation

$$V_c \text{ (plane strain)} \approx V_c \text{ (DM)} \tag{B-6}$$

should give a valid estimate accurate to within a factor of 3-4. In terms of calculating strain rate effects, this uncertainty is allowable and does not introduce significant errors.

The average crack-tip shear strain, γ_c^1, associated with one inclined region, is:

$$\gamma_c^1 \approx \frac{V_c}{\ell} \tag{B-7}$$

where ℓ, the width of the region (see Figure B-1d), can be estimated from the etched zone in Figure B-2:

$$\ell \left(\frac{K}{Y} = 0.26\sqrt{\text{in.}}\right) \approx 0.001 \text{ in.} \tag{B-8}$$

Since the region closest to the crack tip--the shaded region in Figure B-1d--

TABLE B-1. COMPARISON OF PREDICTIONS WITH MEASUREMENTS DERIVED FROM PLANE STRAIN PLASTIC ZONES IN AN Fe-3Si STEEL (Y = 62,000 psi) AS REVEALED BY ETCHING

$\frac{K}{Y}$	ρ		ϵ_c		$\epsilon_{0.3\rho}$	
	measured	calculated(a)	measured(b)	calculated(c)	measured(b)	calculated(d)
0.266(e)	0.008	0.014	>3%	7.3%	~ 1%	0.9%
0.532(f)	0.042	0.056	--	--	--	--

(a) Equation B-5.

(b) Measured values derived from the intensity of the etching response (see Reference 11 for details). At the crack tip the value is corrected for reverse plastic deformation accompanying unloading. The "on-load" value is taken as 2/3 of the strain existing after loading and unloading.

(c) Equation (B-10), ℓ = 0.001 in. (see Figure B-2), $E = 30 \cdot 10^6$ psi.

(d) Equation (B-11), ℓ = 0.002 in. (see Figure B-2), $E = 30 \cdot 10^6$ psi.

(e) See Figure B-2; fatigue-cracked rectangular plate coupon 2-1/2 in. wide by 0.060 in. thick with 0.25 in. long edge cracks.

(f) Rectangular plate coupon 2-1/2 in. wide by 0.406 in. thick with 0.25 in. long edge slots, 0.006 in. wide with a root radius ~ 0.003 in.

represents the superposition of the two inclined regions, the tensile strain ϵ_c here is:

$$\epsilon_c \approx \left(1/2\ \gamma_c^1 + 1/2\ \gamma_c^2\right) \approx \frac{V_c}{\ell} \tag{B-9}$$

Combining Equations (B-9), (B-6), and (B-1a):

$$\epsilon_c \approx \frac{Y}{2\ell E}\left(\frac{K}{Y}\right)^2 \quad . \tag{B-10}$$

Similarly, ϵ_ρ, the strain at some fractional distance $f\rho$ along the inclined region, can be estimated in the same way:

$$\epsilon_\rho \approx \frac{Y}{4\ell_\rho E}\left(\frac{V_\rho}{V_c}\right)\left(\frac{K}{Y}\right)^2 \quad , \tag{B-11}$$

where the ratio V_ρ/V_c has the same significance as the ratio V/V_c calculated in Figure 3 of Reference 11, and ℓ_ρ can be estimated from etched zones. As shown in Table B-1, strain values calculated with these equations are consistent with strain indications derived from the intensity of the etching response of etched plastic zones.

Finally, the crack-tip strain rate, $\dot{\epsilon}_c$, can be derived by differentiating Equation (B-10):

$$\dot{\epsilon}_c \approx \frac{1}{\ell E}\left(\frac{K}{Y}\right)\dot{K} \quad , \tag{B-12}$$

where $\dot{K} \equiv \dot{T}\sqrt{\pi c}$, $\dot{T}$ is the nominal loading rate, Y is yield stress corresponding to $\dot{\epsilon}_c$ and provided ℓ can be assumed to be constant. For values typical for mild steel at low temperatures: K (or K_{Ic}) = 40 ksi$\sqrt{\text{in.}}$, Y = 140 ksi, ℓ = 0.001 in., and E = 30,000 ksi, $\dot{\epsilon}_c/\dot{K} \approx 9.5 \cdot 10^{-3}(\text{ksi}\sqrt{\text{in.}})^{-1}$, and this is surprisingly close to the value $\dot{\epsilon}_c/\dot{K} \approx 2.41 \cdot 10^{-3}(\text{ksi}\sqrt{\text{in.}})^{-1}$ derived by Krafft [4] from elastic considerations.

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An experimental method of identifying theplastic constraint ahead of a sharp crack loaded under plane-strain conditions is proposed. The method is based on the idea that the cleavage stress--which can be measured with unnotched bars--is the peak stress developed ahead of a crack just prior to crack extension. Ways of calculating the strain, strain rate, and yeild stress appropriate for theplastic region just ahead of the crack are developed. The ration of the cleavage stress to the local yield stress identifies the plastic constraint factor at the stress level corresponding to crack extension. Experimental results recently reported by Krafft are shown to be consistent with this interpretation. With these data, the following expression for p.c.f., the plastic constraint factor, is deduced:

$$\text{p.c.f.} = 1 + 2\frac{K}{Y}$$, where Y is

the yield stress, K the stress intensity parameter, and the numerical constant, 2, has the dimensions inches -1/2. This result offers a way of formulating K_{Ic}, the fracture toughness for crack extension by cleavage, in more basic terms and sheds some light on the metallurgical origins of K_{Ic}.

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