

SSC-179

**Residual Strains And Displacements
Within The Plastic Zone
Ahead of A Crack**

by

J. Cammett, A. R. Rosenfield, and G. T. Hahn

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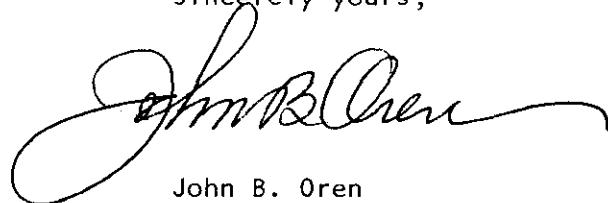
Dear Sir:

Another advance has been made in the measurement of localized yielding around a notch in an experimental study conducted at Battelle Memorial Institute and sponsored by the Ship Structure Committee. "Residual Strains and Displacements within the Plastic Zone Ahead of a Crack" by J. Cammett, A. R. Rosenfield, and G. T. Hahn (SSC-179) describes the interferometric technique used to produce this advance in technology.

The Project has been conducted under the advisory guidance of the National Academy of Sciences - National Research Council, utilizing its Ship Hull Research Committee.

Comments on this report would be welcomed and should be addressed to the Secretary, Ship Structure Committee.

Sincerely yours,

A handwritten signature in black ink, appearing to read "John B. Oren". The signature is fluid and cursive, with a large initial "J" and "O".

John B. Oren
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

SSC-179

Third Progress Report
on
Project SR-164
"Local Strain Measurement"
to the
Ship Structure Committee

RESIDUAL STRAINS AND DISPLACEMENTS
WITHIN THE PLASTIC ZONE
AHEAD OF A CRACK

by

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ABSTRACT

Strains and displacements in the plastically yielded region generated ahead of a machined notch and a crack were detected with an interferometric technique. The measurements were performed on Fe-3Si steel sheets after unloading and reflect local yielding under plane stress conditions. The results show that notch acuity within the limits examined has little effect on the strain distribution. Measured displacements are qualitatively in accord with the theoretical expectations of the DM (Dugdale-Muskhelishvili) model. Quantitative agreement is not obtained and this is attributed to work hardening and the Bauschinger effect, complications that are neglected in the calculation. The work also draws attention to a parameter -- the width of the plastic zone at half maximum strain -- useful for connecting displacement with maximum strain.

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INTRODUCTION

Crack extension in metal sheets and plates is usually preceded by localized yielding and plastic flow. The character of the flow within the plastic zone is important because it influences the stresses and strains generated near the crack tip and, in this way, modifies the conditions for crack extension. Recent studies of an Fe-3Si steel by the authors^(1,2), exploiting an etching technique, have revealed the three-dimensional shape of plastic zones. As shown in Figure 1, the plastic zones consist of two intersecting regions of shear that are wedge-shaped and inclined at 45° to the tension axis. This type of relaxation is observed when the length of the plastic zone is substantially greater than the

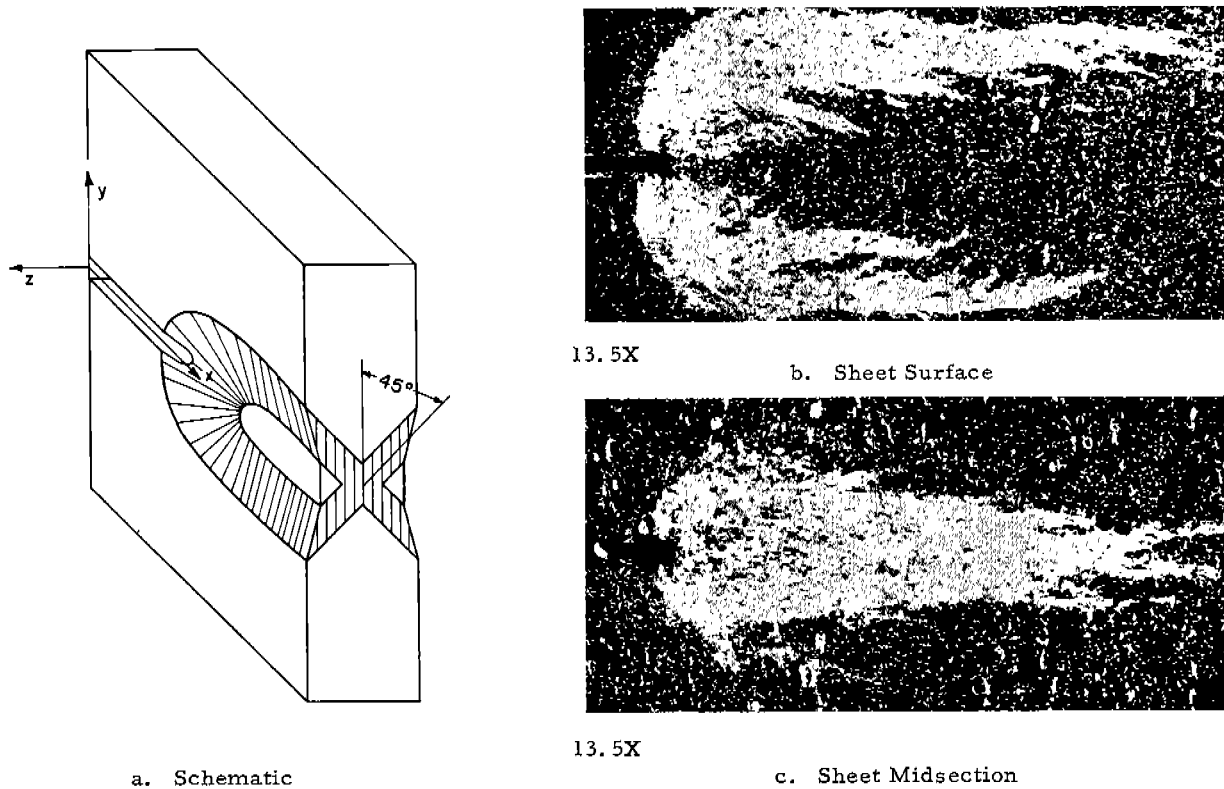


Fig. 1. Views of the Plastic Zone Generated under Plane Stress Conditions (Fe-3Si Steel, $t=0.128$ in., $T/Y=0.90$): Plastic Zones Revealed by Etching Sheet Surface (b) and the Sheet Midsection (c).

plate thickness.† Since the regions of shear completely penetrate the plate, relaxation through the thickness direction is not constrained, $\sigma_z \sim 0$, and a state approaching plane stress prevails.

Under these conditions, the DM (Dugdale-Muskhelishvili) model⁽¹⁻³⁾ -- a crack with a single non-inclined wedge-shaped plastic zone -- simulates the local stress-strain environment. The model offers mathematical expressions of the plastic-zone length, the crack-tip displacement, and the (ductile) crack-extension stress. These have been tested in some instances, and are in reasonable accord with experiment. However, more data are needed to establish the generality of the model and define its limitations. This paper presents more-complete displacement measurements than have been reported previously.⁽¹⁾ The measurements are similar to those of Bateman, et al⁽⁴⁾, on aluminum alloys. However, both their results and those of Dixon and Strannigan⁽⁵⁾ reflect local yielding with a strong plane strain component and are not comparable with the DM model. The present results agree qualitatively with the DM predictions, and are roughly within a factor of 2 of calculated values. In part, the deviations stem from work hardening and the Bauschinger effect -- complications which are not taken into account in the present calculations.

EXPERIMENTS AND CALCULATIONS

The experiments were performed on annealed Fe-3Si steel‡(lower yield stress 62,400 psi) in the form of rectangular coupons (8 inches long by 2.5 inches wide) in 3 thicknesses: 0.200, 0.058, and 0.017 inch. The test coupons were prepared with two centrally located edge notches, either a 1/4-inch-long x 0.006-inch-wide machined slot or a 1/4-inch-long fatigue crack. The notched coupons were then

† When $\frac{c}{t} > 4 \left(\sec \frac{\pi T}{2Y} - 1 \right)^{-1}$ where $2c$ is the crack length, t is the plate thickness, T is the net section stress, and Y is the yield stress of the material.

‡ This material, which was selected because of its unique etching characteristics, displays a stress-strain curve whose shape is very similar to plain carbon steel.^(1,2)

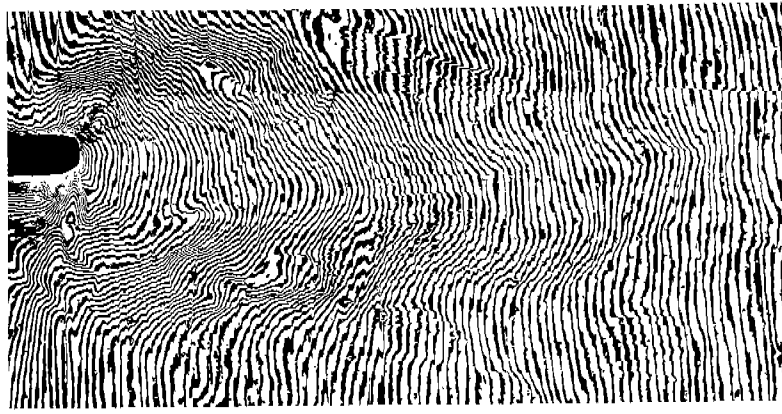
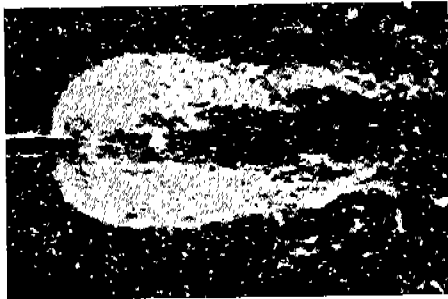
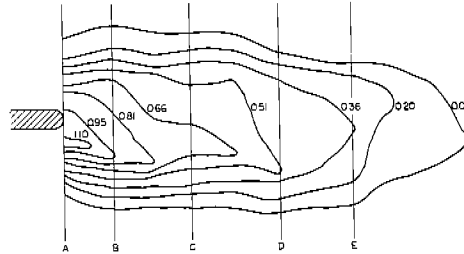


Fig. 2. Interferometric Fringe Pattern of a Plastic Zone in Machine-Notched Fe-3Si Sample ($t=0.058$ in., $T/Y=0.78$).



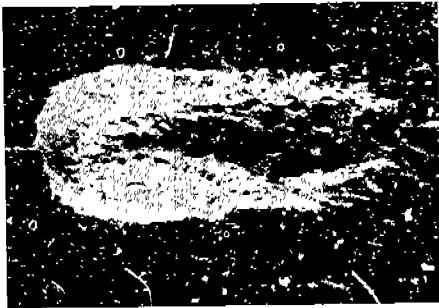
10 X

a. Machine Slotted



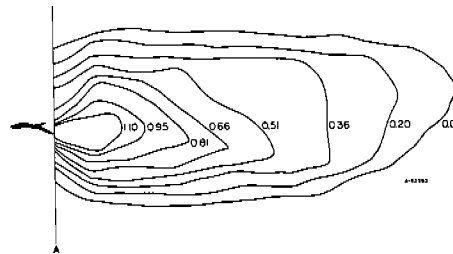
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b. Machine Slotted



10 X

c. Fatigue Cracked



10 X

d. Fatigue Cracked

Fig. 3. Influence of Notch Acuity on the Strain Distribution in the Plastic Zone (Fe-3Si Steel, $t=0.050$ in., $T/Y=0.78$): Plastic Zone (a and c) Revealed by Etching, and Transverse Strain Field (ϵ'_2) (b and d) Derived from Interferometric Fringe Pattern.

slowly loaded to various peak loads in a tensile-testing machine. This load was maintained for 4 minutes and then gradually released. Stress levels are reported either as T, the net section stress, or as T/Y, the ratio of net section stress to yield stress.

After unloading, residual strains† normal to the sheet surface (ϵ'_z , see Figure 1a) were detected and recorded with the aid of an interference microscope.⁽¹⁾ An example of an interferometric fringe pattern is reproduced in Figure 2. The iso-strain contours derived from typical fringe patterns are shown in Figure 3. These contours reflect the residual transverse-strain field $\epsilon'_z (z = 0)$. A series of vertical sections through the strain field (see Figure 3a) are presented in Figure 4. To the extent that the plastic deformation is confined to shear within the 45°-inclined wedge and is large compared with the elastic strains, the approximations $\epsilon'_z \approx \epsilon'_y$ and $\epsilon'_x \approx 0$ are valid. In that case, v'_y , the residual displacement in the longitudinal direction (arising from plastic deformation within the zone) can be obtained from the interferometric measurements of the transverse strain:

$$v'_y = -1/2 \int \epsilon'_z dy \quad . \quad (1)$$

The quantity v'_y is important because it is defined by the DM model, and thus serves as a link between theory and experiment. The value of v'_y at any distance (x-c) from the crack tip is simply 1/2 the area under the appropriate $\epsilon'_z - y$ curve of the type shown in Figure 4. The value of $v'_{y(x=c)} \equiv v'_c$ is referred to as the residual crack-tip displacement.

The simple DM formulations describe a crack under load; expressions for v'_y and v'_c are given in the Appendix. However, the theory can be extended to take into account the relaxations accompanying unloading. If this is accomplished by

† To differentiate between "on-load" values of strain and displacement, and the residual values existing after the load is released, the latter are designated by prime marks, i.e., ϵ'_z , v'_y , v'_c . The change in displacement during unloading is denoted by a double prime mark, i.e., $v'' = v - v'$.

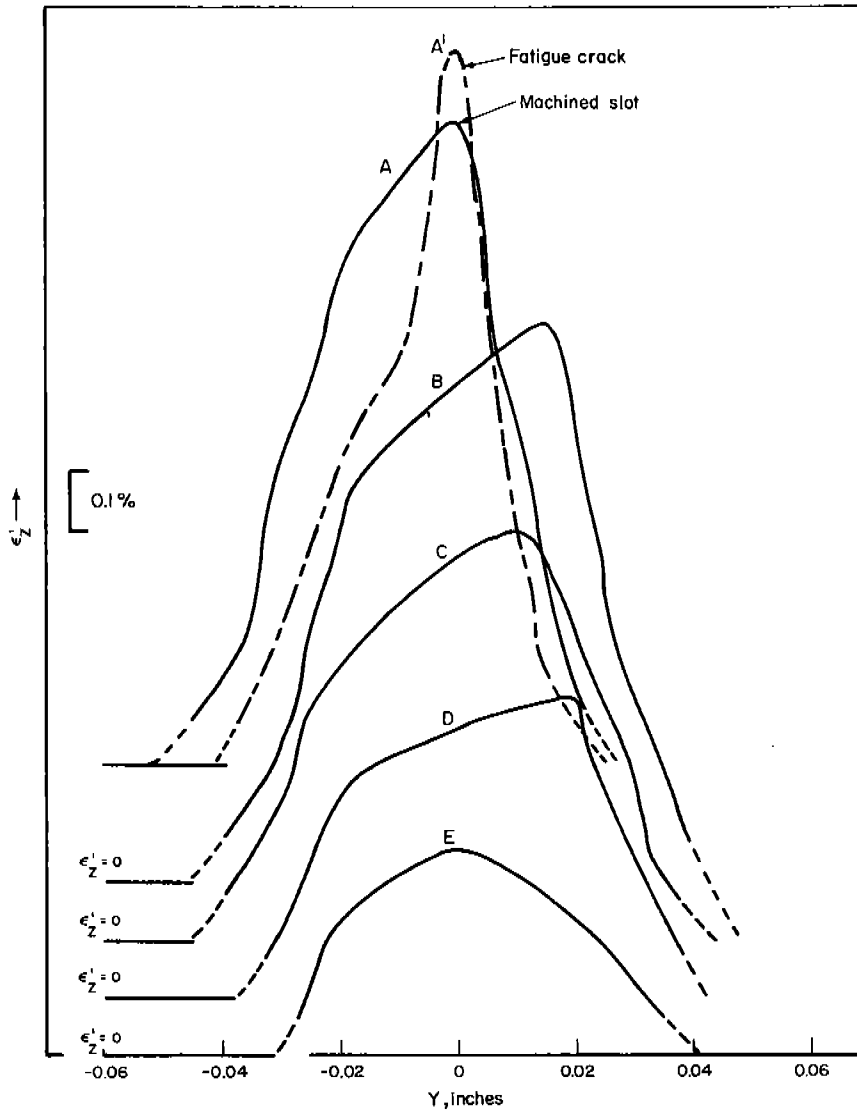


Fig. 4. Longitudinal Sections A, A', B, C, D, and E [See Figures 3(b) and 3(d)] Through the Transverse Strain Field. Note: Different Origins used in Order to Separate Curves.

removing the stress singularity at the plastic-zone tip, then:

$$v_c' = 1/4 v_c \quad (2)$$

at low values of the applied stress.⁽¹⁾ An alternate derivation by Hult and McClintock⁽⁶⁾ -- more consistent with the superposition principle -- removes the singularity at the crack tip. The calculation, recently discussed by Rice⁽⁵⁾ and outlined in the Appendix, describes the entire displacement gradient. It yields larger residual values; at low stresses,

$$v_c' = 1/2 v_c \quad (3)$$

Experimental support for Equation (3) has been reported in austenitic steel and copper by Dixon and Strannigan⁽⁵⁾ under conditions of plane strain and by Bateman, et al⁽⁴⁾, in aluminum under conditions of mixed plane strain and plane stress. Dixon and Strannigan's⁽⁵⁾ results on brass ($v_c' \approx 1/6 v_c$) are closer to the prediction of Equation (2).

In addition to displacement, it is important to know the strains within the plastic zone and particularly at the crack tip, since any criterion for ductile fracture is likely to be phrased in terms of the local strain. Strain values can be calculated from the width and cross section of the plastic zone. A simple model representing the cross section of the zone as a double trapezoid has been described by Rosenfield, et al⁽²⁾. They have shown that

$$\bar{\epsilon} \sim \frac{4v_y}{d} , \quad (4)$$

where $\bar{\epsilon}$ is the maximum value of true strain at any distance $(x-c)$, $y > 0$ from the crack tip, v_y is the displacement at that point, and d is the zone width. As a practical matter, it is often inconvenient to measure d , since the zone boundary is not sharp. The problem is further complicated if the sample bends. For symmetrical zones such as observed in Fe-3Si, it is more convenient to measure δ , the width of the zone at half-maximum transverse strain. According to the earlier model, $\delta = \frac{d}{2}$ and Equation (4) becomes

$$\bar{\epsilon} \sim \frac{2v_y}{\delta} . \quad (5)$$

Other descriptions of the zone cross section lead to about the same numerical term of Equation (5). For example, use of a Gaussian curve will change the numerical factor in Equation (5) from 2.0 to 1.89.

RESULTS AND DISCUSSION

Results obtained for the fatigue crack and the more blunt machined slot

are summarized in Figures 4 to 8. Both the etch patterns (Figure 3) and the interferometric measurements show that the two notches generate plastic zones nearly identical, in terms of shape, size, and the strain distribution within. Some differences in the strain distribution persist at distances very close to the notch root, e.g., distances comparable with the notch root radius. Unless the notch is very blunt, this region will not interact with the 45° -inclined wedges. It seems likely that the shear within the 45° -inclined wedges is crucial in ductile crack extension. Consequently, if the results obtained here are general, then notch acuity (within the limitation mentioned) may have only a minor effect on crack extension under plane stress.

Figures 5 and 6 illustrate that the residual displacement gradients and crack-tip values are qualitatively in accord with the DM model. The residual crack-tip values (Figure 7) are described very well by Equation (2), as found previously⁽¹⁾, and while this expression is therefore useful, the agreement is probably fortuitous. A closer examination of the problem reveals four complicating features which lead to an overestimation of v_y^* by the calculations and an underestimation by the measurements:

1. Work Hardening. The present calculations do not take into account the work hardening accompanying plastic deformation. Work hardening alone, in the absence of a Bauschinger effect, progressively increases the resistance to flow within the zone both during the loading and the unloading cycle. This has the effect of reducing plastic-zone length and the displacement values v_y and v_y^{**} . In this way, work hardening can account for large discrepancies near the plastic-zone tip (see Figures 5 and 6). Its effect near the crack tip is difficult to evaluate because the residual value is the difference of the two diminished displacements, $v_y^* = v_y - v_y^{**}$.

2. Bauschinger Effect. Another complication neglected in the calculations is the Bauschinger effect, whereby the resistance to flow is diminished after a reversal of the loading. In the face of a Bauschinger effect, the calculations

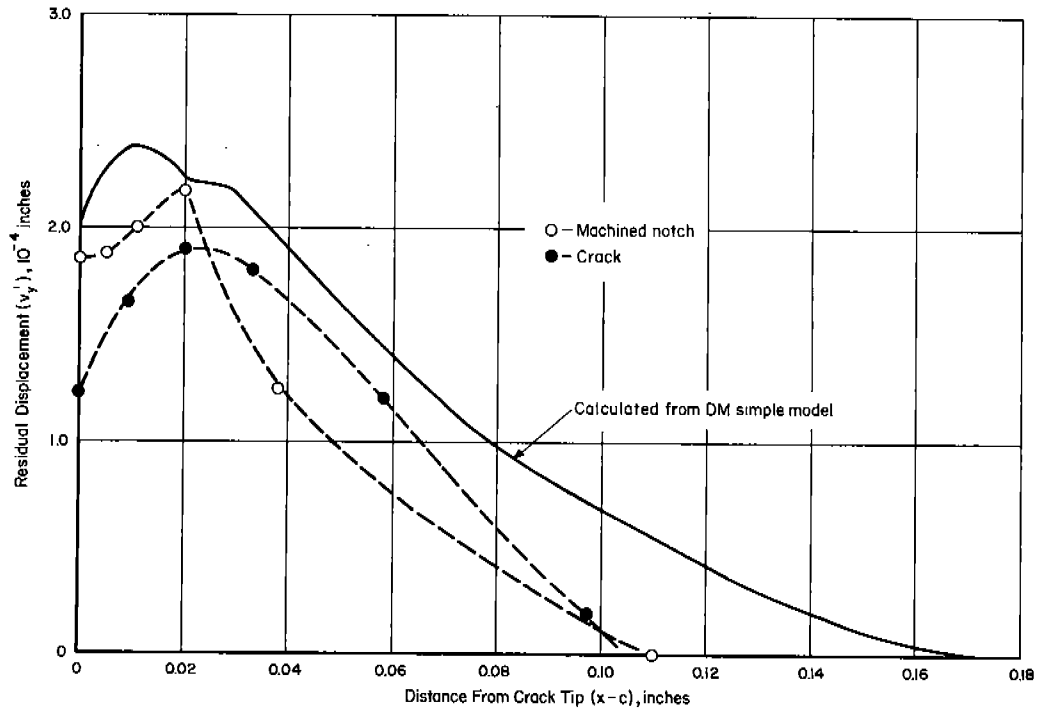


Fig. 5. Measured and Calculated Residual-Displacement Gradients Generated Ahead of a Machined Notch and a Crack ($t=0.017$ in., $T/Y=0.60$).

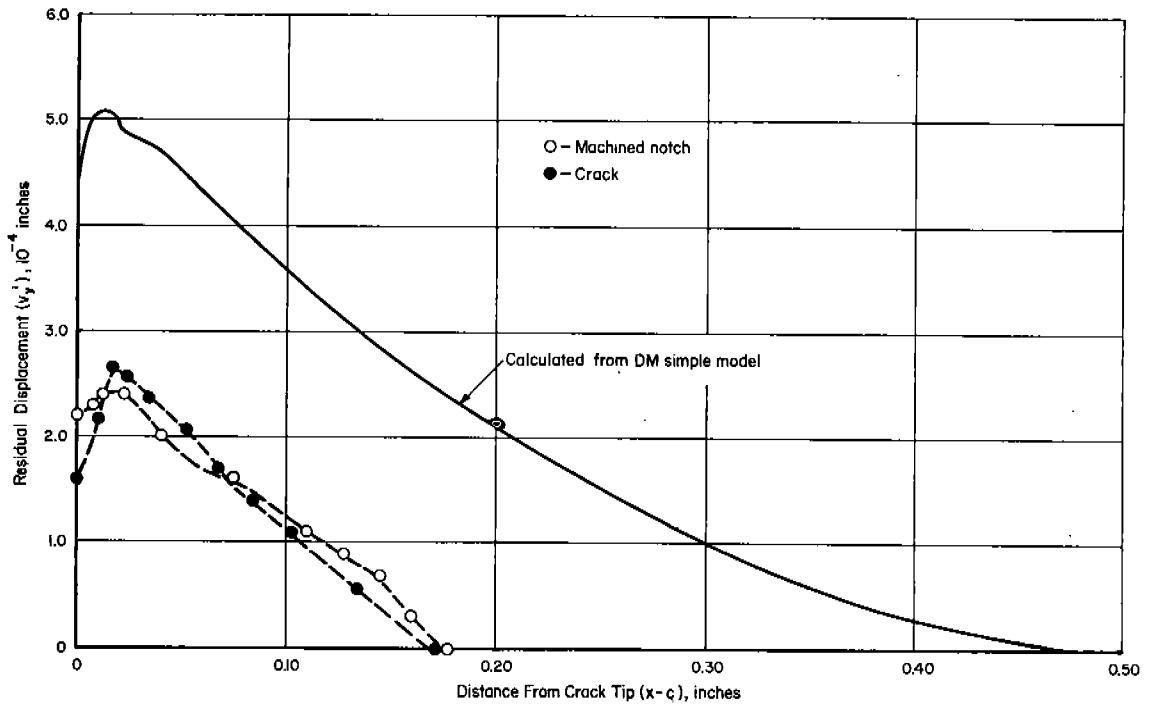


Fig. 6. Measured and Calculated Residual-Displacement Gradients Generated Ahead of a Machined Notch and a Crack ($t=0.058$ in., $T/Y=0.78$).

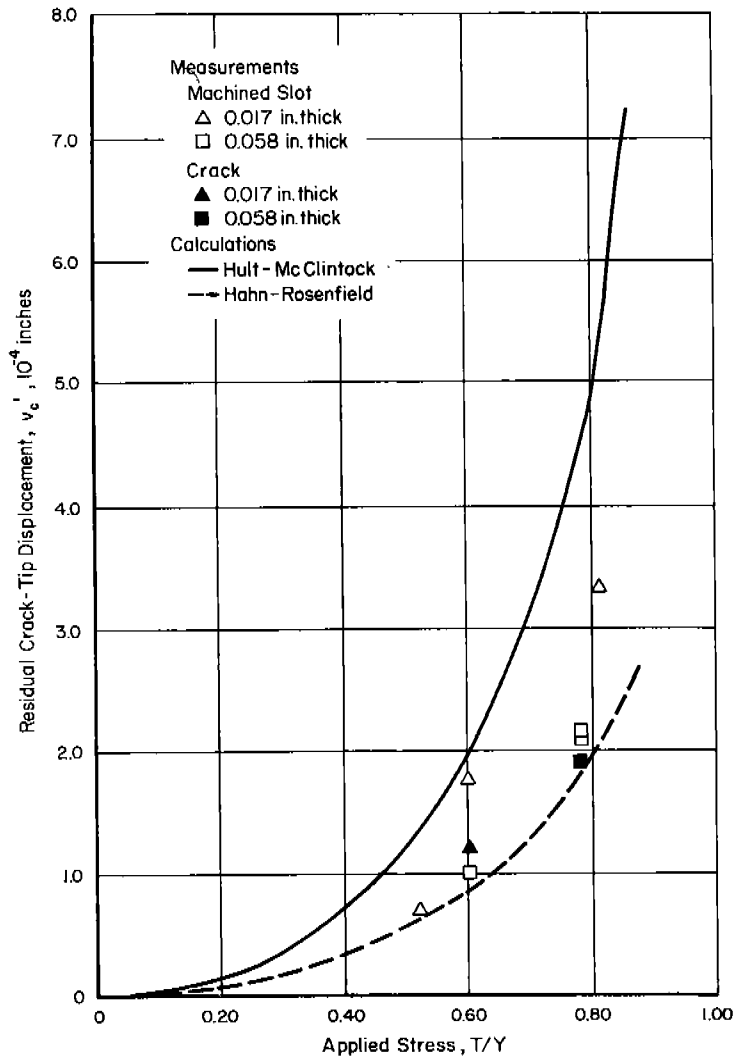


Fig. 7. Stress Dependence of Measured and Calculated Residual Crack-tip Displacement Values v'_c .

overestimate the value of Y appropriate for the unloading cycle, underestimate v_y'' (see Equation A-5), and thus overestimate v_y' .

3. Residual Stresses. One of the assumptions implicit in the DM model is that the material within the plastic zone is rigid plastic. Calculated values of v_y , v_y'' , and v_y' reflect only those displacements arising from plastic deformation; the contribution of elastic strain within the zone is neglected. This is a reasonable assumption because the DM zone is very narrow; the stresses acting on it are comparable to the yield stress, and the elastic displacements are therefore small. Real plastic zones tend to be much wider than the DM zone, and the elastic contribution can become a significant part of the total displacement across the plastic zone.

The longitudinal displacements quoted here were obtained from measurements of the transverse strains near the crack tip. The transverse elastic contribution is approximately

$$v'_{z(e)} \approx \frac{Y \nu d}{E} , \quad (6)$$

where ν is Poisson's ratio and E is the modulus. For the Fe-3Si samples, $d \approx 1.7t$, and the resulting displacements are $v'_{z(e)} \approx -0.2 \cdot 10^{-4}$ inch and $v'_{z(e)} \approx -0.7 \cdot 10^{-4}$ inch for the 0.017-inch- and 0.008-inch-thick samples, respectively. Together with Figure 7, these results indicate that this elastic contribution is negligible at all but very small stress levels.

4. Plane-Strain Relaxation. The analysis of the measurements depends on the assumption that none of the plastic deformation is in the plane of the sheet -- but this is only an approximation. Near the crack tip and to a lesser extent elsewhere, some flow in the plane of the sheet is likely, and this is not detected by the interferometric technique. For this reason, the measurements tend to underestimate the residual displacement values.

It is difficult to gage the cumulative effect of these errors at this time; however, a rough estimate suggests that work hardening alone can probably account for a large part of the discrepancy between the measured values and the DM calculation employing the Hult-McClintock analysis.⁽⁶⁾ The Hult-McClintock treatment, therefore, appears to be sound. It can be improved by correcting for work hardening and the Bauschinger effect along the lines already proposed.⁽²⁾ In this form the analysis may offer a useful description of the residual displacement gradient under plane-stress conditions, not only after one cycle, but after repeated cycles of loading and unloading. These possibilities are now being studied.

The correction for work hardening and Bauschinger effect would be facilitated by a simple relation between strain and displacement. Equation (5) could satisfy this need, but only if the quantity δ -- related to the work hardening rate -- is a materials constant, or provided δ is not a sensitive function of

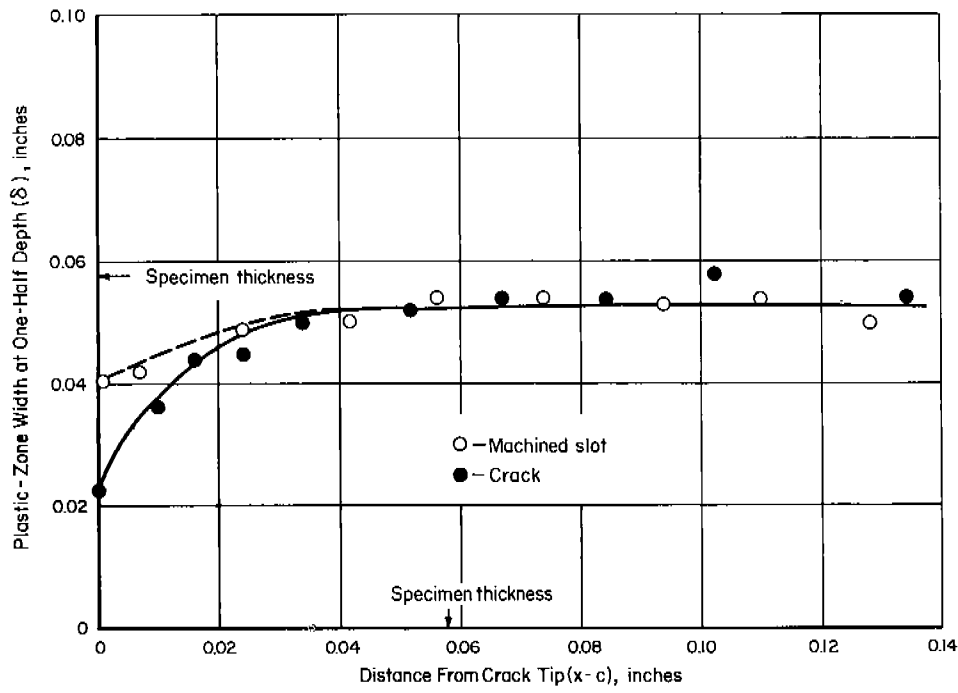


Fig. 8. Plastic-Zone-Width Parameter (δ) Versus Distance from Crack Tip.

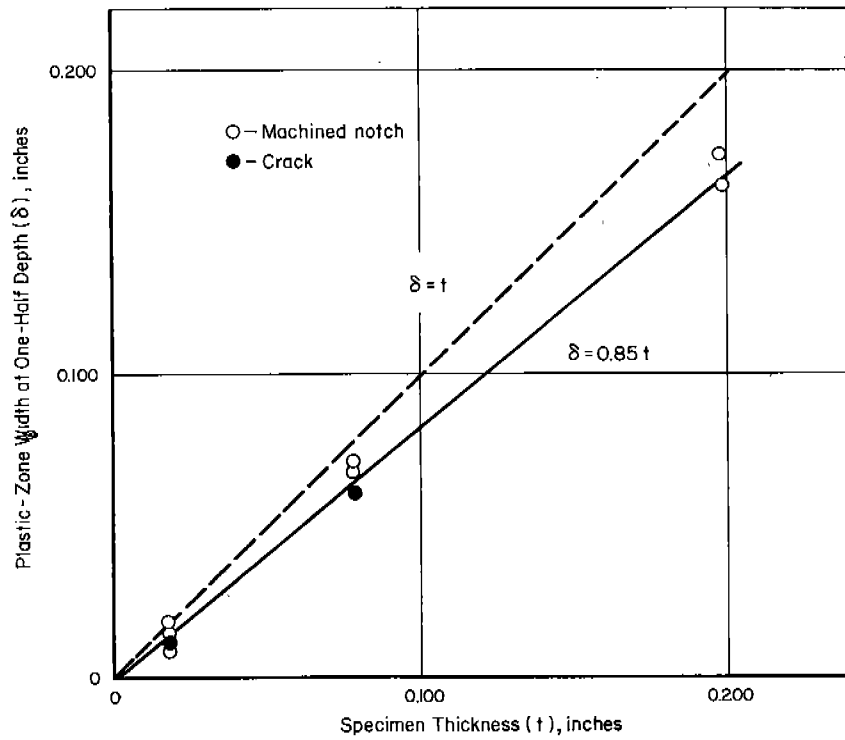


Fig. 9. Influence of Sheet Thickness at Different Stress Levels on the Plastic Zone Width Parameter (δ) Measured at a Fixed Distance, $x - c = t/2$, from the Crack Tip.

stress level and geometry.

The results of Figures 8 and 9 show that at least for Fe-3Si δ is essentially independent of distance from the crack tip, that it can be correlated with thickness, and that it is not a strong function of stress. Since $\delta = 0.85 t$, where t is the sheet thickness, as shown in Figure 9, Equation (5) becomes:

$$\epsilon'_y = \frac{2.3}{t} v'_y \quad (7)$$

This gives hope that the appropriate δ value for different materials can be established with a small number of measurements.

CONCLUSIONS

1. Residual displacement values in advance of notches and cracks arising from localized plastic deformation under plane stress conditions are described qualitatively by the DM model, together with the Hult-McClintock method of treating unloading. Quantitative agreement is not obtained and this is attributed mainly to work hardening and the Bauschinger effect, complications that are neglected in the calculations.

2. Notch acuity, within the narrow limits examined here, has relatively little effect on the strains and displacements generated under plane stress conditions. The expectation is that notch acuity, under the same conditions, exerts a minor influence on crack extension.

3. The plastic zone width at half maximum strain, appears to be a useful parameter for relating the maximum strain with the displacement under plane stress conditions. The present measurements show that for Fe-3Si, this quantity varies linearly with plate thickness independent of stress level, and is fairly constant along the length of the plastic zone.

ACKNOWLEDGMENTS

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APPENDIX

CALCULATION OF RESIDUAL DISPLACEMENT

According to a procedure originally discussed by Hult and McClintock⁽⁶⁾, and employed by other authors^(5,7), unloading can be represented by application of a compressive stress to a material with a yield stress equal to twice the initial yield stress. According to the DM model, the on-load displacement at the crack tip, v_c , is given by

$$v_c = \frac{4Yc}{\pi E} \ln \sec \beta \quad , \quad (A-1)$$

where Y is the yield stress, $2c$ is the crack length, E is Young's modulus, and $\beta = \frac{\pi T}{2Y}$, with T the applied stress. Using the Hult-McClintock procedure, the change in crack-tip displacement on unloading, v_c'' , is found by replacing Y in Equation (A-1) by $2Y$,

$$v_c'' = \frac{8Yc}{\pi E} \ln \sec \frac{\beta}{2} \quad . \quad (A-2)$$

The residual crack-tip displacement, v_c' , is then equal to $v_c - v_c''$,

$$v_c' = \frac{4Yc}{\pi E} \ln \frac{1 + \sec \beta}{2} \quad . \quad (A-3)$$

Similarly, the displacement at any point in the plastic zone, $v(x)$, is given by

$$v(x) = \frac{2(c + \rho)Y}{\pi E} \left[\cos \theta \ln \frac{\sin(\beta - \theta)}{\sin(\beta + \theta)} + \cos \beta \ln \frac{\sin \beta + \sin \theta}{\sin \beta - \sin \theta} \right] \quad , \quad (A-4)$$

where ρ is the plastic-zone length, $\cos \theta = \frac{x}{(c + \rho)}$, and $\cos \beta = \frac{c}{(c + \rho)} = \frac{\pi T}{2Y}$.

The change in displacement on unloading, $v(x)''$, is found by replacing Y in

Equation (A-4) with $2Y$. This gives rise to new values: $\beta'' = \frac{\beta}{2}$,

$\rho'' = c(\sec \beta'' - 1)$, and $\cos \theta'' = \frac{x}{c + \rho''}$:

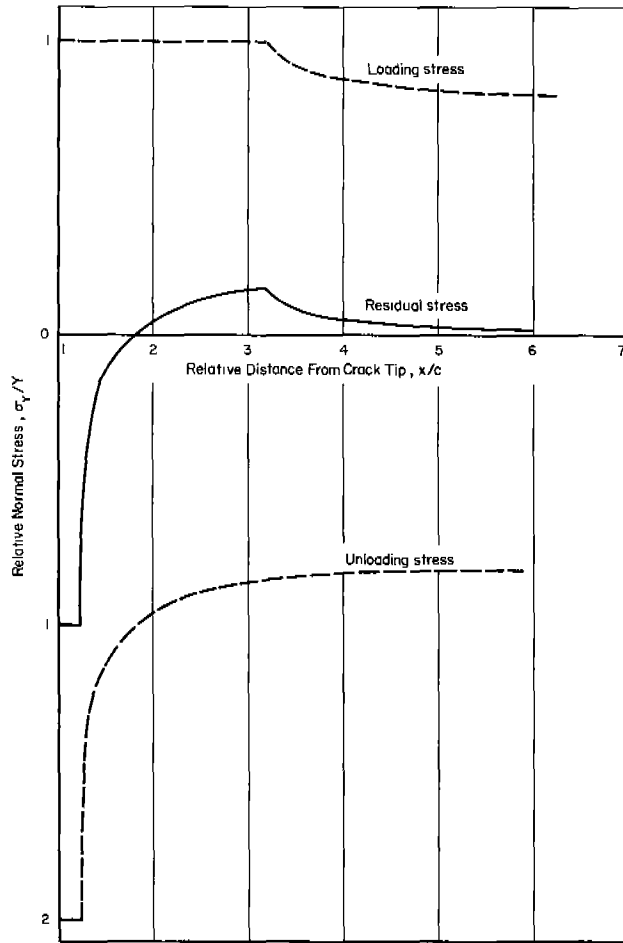


Fig. A-1. Stress Distribution in Front of a Crack for $T/Y=0.8$ According to the DM Model.

$$v(x)^* = \frac{2(c + \rho^*)Y}{\pi E} \left[\cos \theta \ln \frac{\sin(\beta^* - \theta^*)}{\sin(\beta^* + \theta^*)} + \cos \beta^* \ln \frac{\sin \beta^* + \sin \theta^*}{\sin \beta^* - \sin \theta^*} \right] \quad (A-5)$$

Equation (A-5) is valid in the range $(c + \rho^*) \geq x \geq c$. Thus, reverse plastic flow is confined to the small fraction of the plastic zone given by

$\rho^*/\rho = \frac{\sec(\beta/2) - 1}{\sec \beta - 1}$. The ratio $\rho^*/\rho \approx 1/4$ at low stresses and decreases toward zero at high stresses.

As before, the residual displacement $v(x)^*$ is equal to $v(x) - v(x)^*$. Since $v(x)^*$ is a large fraction of $v(x)$ close to the crack but falls off very rapidly, a maximum will be observed in the curve of $v(x)^*$ versus x .

The residual stress distribution after unloading can be calculated in a similar manner. The stress at any point in front of a crack, σ_y , is given by

$$\frac{\sigma_y}{T} = 1 + \frac{1}{\beta} \arctan \frac{\sin 2\beta}{e^{2\alpha} - \cos 2\beta} ; x > \frac{c}{\cos \beta} \quad (A-6)$$

$$\sigma_y = Y \quad ; \quad \frac{c}{\cos \beta} > x > c$$

where $\cosh \alpha = \frac{x \cos \beta}{c}$. The stresses on unloading, σ_y^* , can be found by substituting $2Y$ for Y and $\frac{\beta}{2}$ for β in Equation (A-6), as was done above. The residual stress, σ_y^* , is then the difference between σ_y and σ_y^* as is illustrated in Figure A-1 for $T/Y = 0.8$. The general features of the stress distribution are: a region very close to the crack tip which yields in tension during loading and compression during unloading, a region which yields during loading but strains elastically during unloading (the residual stress varying from $-Y$ to $+\frac{Y}{6}$), a region of residual tensile stress which strains elastically both during loading and unloading. Dixon's⁽³⁾ calculations show similar results.

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<p>Strains and displacements in the plastically yielded region generated ahead of a machined notch and a crack were detected with an interferometric technique. The measurements were performed on Fe-3Si steel sheets after unloading and reflect local yielding under plane stress conditions. The results show that notch acuity within the limits examined has little effect on the strain distribution. Measured displacements are qualitatively in accord with the theoretical expectations of the DM (Dugdale-Muskhelishvili) model. Quantitative agreement is not obtained and this is attributed to work hardening and the Bauschinger effect, complications that are neglected in the calculation. The work also draws attention to a parameter -- the width of the plastic zone at half maximum strain -- useful for connecting displacement with maximum strain.</p>		

14. KEY WORDS	LINK A		LINK B		LINK C	
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